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Fast Response, Small Ripple, Low Noise Switching Converter with Digital Charge Time Control and EMI Harmonic Filter

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Abstract. This paper describes a selection method of L, C parameters for a given load resistance R to realize well-balanced performance of a buck converter and its EMI harmonic filter design method based on the modern optimal and adaptive control theory. We show that when $|Z_L| = |Z_C| = 2R$ is satisfied at the angular frequency of $f = 1/2\pi\sqrt{LC}$, then fast response, small overshoot and low ripple (smaller than 0.4mVpp) can be realized simultaneously. Also by adopting two LC notch filters whose notches are at the fundamental and second harmonic frequencies respectively, the EMI noises are reduced significantly (7dB reduction at the 1st harmonic, 2dB reduction at the 2nd harmonic).

1. Introduction

A typical simplified network of a switching voltage converter is shown in Fig. 1. A step-down switching regulator controls the output voltage by controlling the duty cycle to a transistor. The duty cycle changes depending on the load requirement. Moreover, the requirements for the EMI noises, the time response, the overshoot phenomena and the output ripple of these systems are extremely strict in recent applications [1, 2, 3].

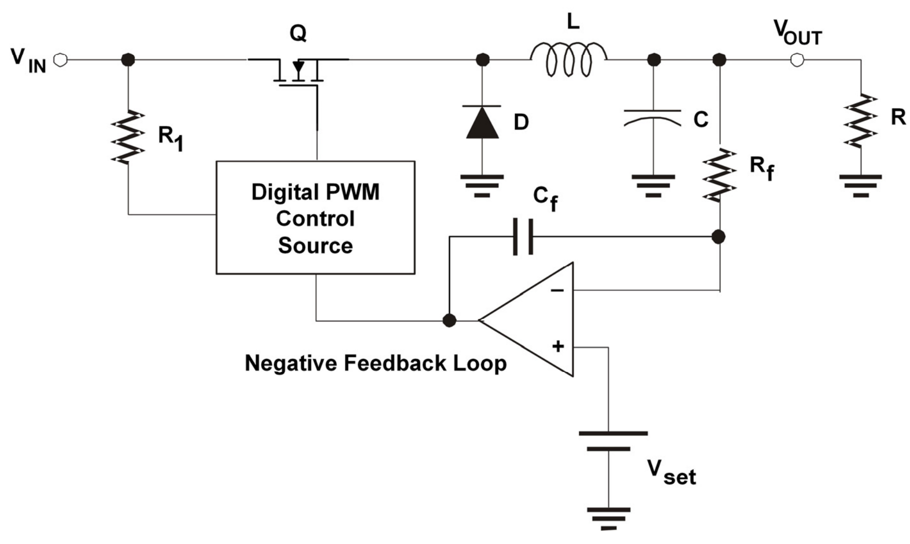


Fig.1. Step-down switching power converter.

2. Background of Switching Analog Signal Processing

2.1 Energy Propagation Principle

In an electronic system, a superposition of energy at one place is proportional with their sources and transmission space [4]. Let $E_A(t)$ be the sum energy of multi-sources $E_i(t)$ which are transmitted on the different resistance distances d_i. The superposition principle for a direct resistance distances can be defined by

$$E_A(t)\sum_{i=1}^{n}\frac{1}{d_i}=\sum_{i=1}^{n}\frac{E_i(t)}{d_i} \tag{1}$$

Energy is propagated in the form of waves.

$$E(d,t)=E_0e^{-\alpha d}\cos(\omega t-\beta d+\phi) \tag{2}$$

The spatial period of the wave is known as the wavelength, and denoted as λ. On the electronic propagation of lumped components (resistors, capacitors, inductors), the time delays (phase) of waves are neglected. However, the length of a line in the distributed elements (transmission lines) which is significantly compared with a wavelength makes the time delays (phase change).

2.2 Switching Regulator

A switching regulator maintains an essentially constant output voltage with changing input voltage. In other words, the energy is converted from the input source into the average output energy in a unit of time. An increase in the duty cycle increases the output voltage, whereas a decrease in the duty cycle decreases the output voltage as shown in Fig. 2.

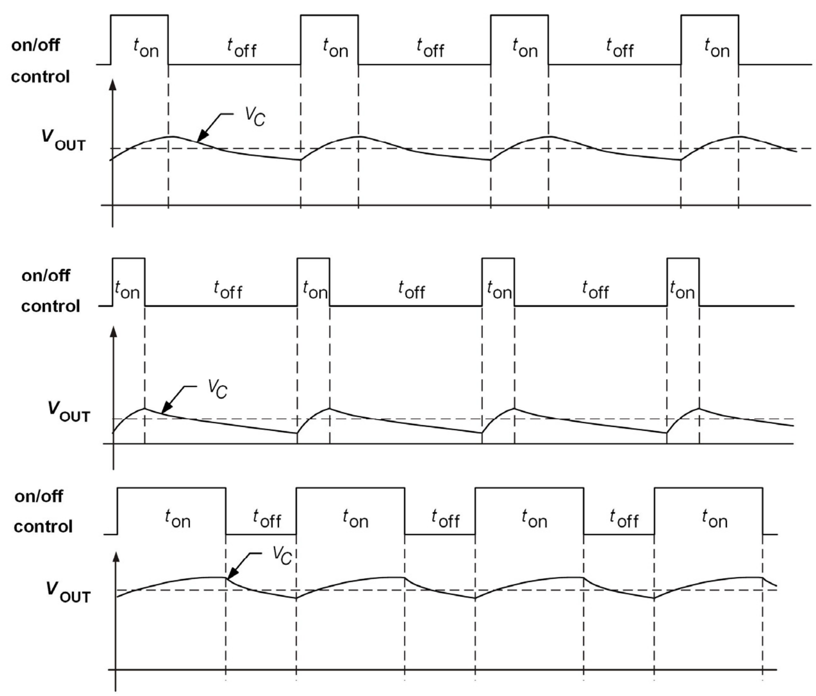


Fig.2. Waveforms of switching sources and output voltages.

The average voltage at the output of the power conversion is defined as

$$\overline{V_{out}}=V_{\text{in}}\left(\frac{T_{\text{on}}}{T_{\text{on}}+T_{\text{off}}}\right) \tag{3}$$

The main control element of this system is a switch which controls the output voltage by rapidly switching the input voltage on and off with a duty cycle that depends on the load. The switching control source is constantly driven back and forth between saturation and cutoff.

2.3 Waveforms and Spectrums of Switching Control Source

Pulse Width Modulation (PWM) signals are widely used in the switching control systems. The waveforms of PWM are also expressed in many functions of time with many different frequencies which are constituted of the times of a fundamental frequency (or a unit cycle T= t_{on}+ t_{off}) as shown in Fig. 3.

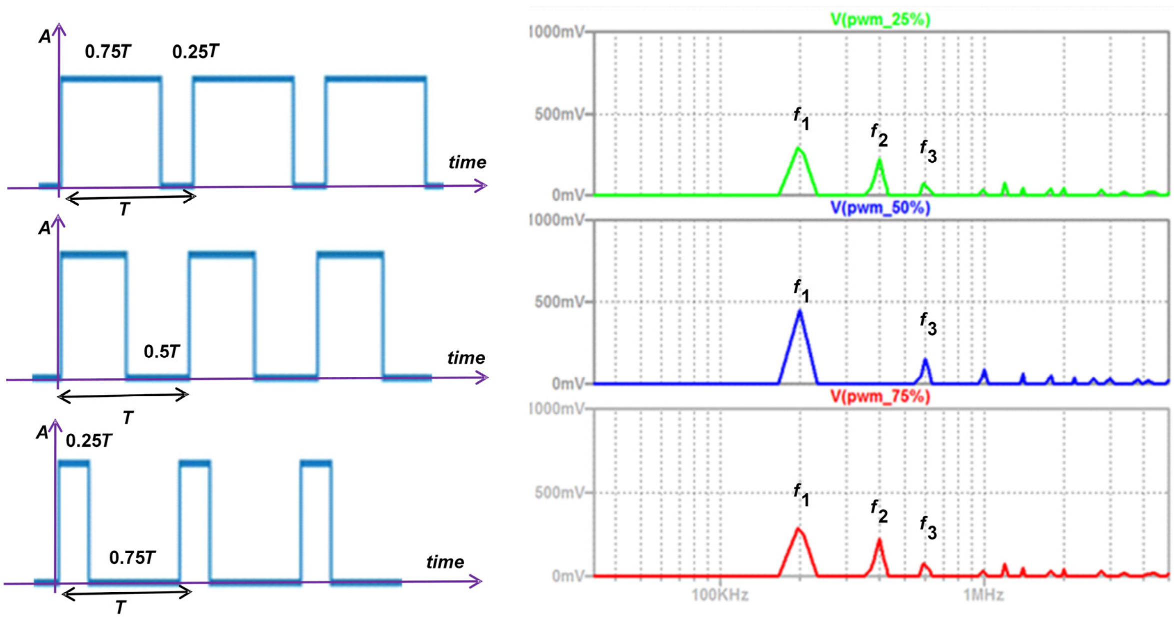


Fig.3. Waveforms and spectrums of switching control signals.

In the Fourier transform principles, some waveforms of these sources are plotted in the frequency domain. Some waveform functions of the control sources are expressed in Eq. (4), Eq. (5), Eq. (6).

$$S_{25\%PWM}(t)=\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi\left(2k-1\right)\left(f_1\right)t\right)}{2k-1}-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi\left(2k-1\right)\left(2f_1\right)t\right)}{2k-1} \tag{4}$$

$$S_{50\%PWM}(t)=\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi\left(2k-1\right)\left(f_1\right)t\right)}{2k-1} \tag{5}$$

$$S_{75\%PWM}(t)=\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi\left(2k-1\right)\left(f_1\right)t\right)}{2k-1}+\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi\left(2k-1\right)\left(2f_1\right)t\right)}{2k-1} \tag{6}$$

3. Analysis of Switching Power Conversion Network

3.1 Simplified Model of Step-down Switching Converter

In analog signal processing, any sources are expressed as a wave function. Based on the switching frequency of the control signal, the input supply source is transferred from a DC supply source into an AC supply signal which contains a lot of harmonics. A simplified model of a switching buck converter is redrawn in Fig. 4.

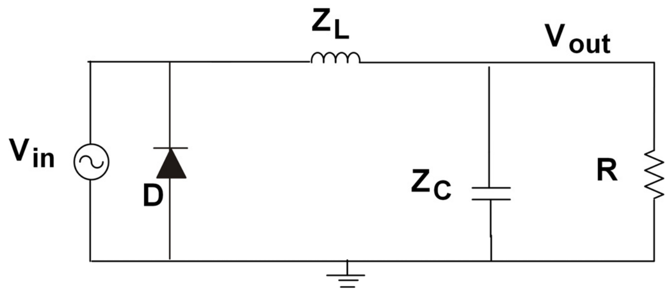


Fig.4. Simplified switching power conversion system.

Assume that $\omega = 2\pi f = 2\pi / T$ is the angular frequency of the current flowing on the above loops. Then, the impedances of the inductor and the capacitor are denoted as Z_L and Z_C in frequency domain $\left(Z_L = j\omega L; Z_C = {}^{1}\!/_{j\omega C}\right)$. In this electro-dynamic system, the modern adaptive control theories are used to value the quality of a network. Therefore, the frequency response at the output node is used to clarify the characteristics of this network in the forms of the Bode plot.

3.2 Frequency Response of RLC Network

A frequency response of a network is used to describe the relationships between the input sources and the output sources in the view of the spectrum energies on the frequency domain. Let us employ the superposition principle at the output node, the voltage relationships between the input sources and the output sources are expressed as

$$V_o\left(\frac{1}{Z_L}+\frac{1}{Z_C}+\frac{1}{R}\right)=\frac{V_{in}}{Z_L} \tag{7}$$

The function of the output voltage is rewritten as

$$V_o = V_{in}\frac{RZ_C}{R\left(Z_L+Z_C\right)+Z_L Z_C} \tag{8}$$

The transfer function of this system is

$$H=\frac{V_o}{V_{in}}=\frac{R_L Z_C}{R_L\left(Z_L+Z_C\right)+Z_L Z_C} \tag{9}$$

Let us modify Eq. 9 into the angular frequency variable form

$$H\left(j\omega\right)=\frac{\frac{1}{LC}}{\left(j\omega\right)^2+j\omega\frac{1}{RC}+\frac{1}{LC}} \tag{10}$$

Then, the denominator is modified as

$$H\left(j\omega\right)=\frac{\frac{1}{LC}}{\left(j\omega\right)^2+2j\omega\frac{1}{2RC}+\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2} \tag{11}$$

Let us reorganize Eq. (11) as

$$H\left(j\omega\right)=\frac{\frac{1}{LC}}{\left(j\omega+\frac{1}{2RC}\right)^2+\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2} \tag{12}$$

To transfer the maximum power, the denominator of Eq. (12) has to be maximal. So, the constrained condition is

$$\frac{1}{LC}-\left(\frac{1}{2RC}\right)^2=0 \tag{13}$$

In addition, in the LC loop, the angular frequency is $\omega = 1/\sqrt{LC}$; Eq. (13) is rewritten as

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{2RC} \tag{14}$$

The values of R, L, and C are chosen based on the following relation

$$|Z_L|=|Z_C|=2R \tag{15}$$

Then, in Eq. 12, the magnitude of the transfer function is rewritten as

$$H(j\omega)=\frac{\frac{1}{LC}}{\left(j\omega+\frac{1}{2RC}\right)^2} \tag{16}$$

To plot the Bode graph of the magnitude of the transfer function is rewritten as

$$|H(\omega)|=\frac{\frac{1}{LC}}{\left(\frac{1}{2RC}\right)^2+\omega^2} \tag{17}$$

The cutoff frequency is defined as

$$\omega_{cut\_off}=\frac{1}{\sqrt{LC}}=\frac{1}{2RC} \qquad or \qquad f_{cut\_off}=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{4\pi RC} \tag{18}$$

The maximum power is transferred at the cutoff frequency

$$|H(\omega)|=\frac{1}{2} \qquad or \qquad |H(\omega)|_{dB}=20\log(\frac{1}{2})=-3dB \tag{19}$$

In addition, the average energy of the capacitor is defined by the charge and discharge energy of the capacitor (or the differential potential voltages between two plates of the capacitor) in every unit time. The charge and discharge voltages of the capacitor in one cycle are dependent on the initial level voltage of the capacitor and the duty ON, duty OFF of PWM. To have a stable energy on the capacitor, the charge and discharge time of the RC loop and the LC loop has to be the same value. In the LC loop, the charge and discharge time is very fast. So the energy is transferred very fast for a short time.

- In case $|Z_L|=|Z_C|>2R$, the charge time of the capacitor is smaller than the discharge time.
- In case $|Z_L|=|Z_C|=2R$, the charge time of the capacitor is equal to the discharge time.
- In case $|Z_L|=|Z_C|<2R$, the charge time of the capacitor is greater than the discharge time.

When the maximum energy propagation satisfies $|Z_L|=|Z_C|=2R$, the overshoot phenomena are perfectly controlled. In this system, the cutoff frequency is designed at

$f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{4\pi RC} = 5kHz$ (R = 5Ω, L = 318μH, C = 3.18μF). When the charge time of the capacitor is greater than the discharge time, the overshoot is a big problem as shown in Fig. 5.

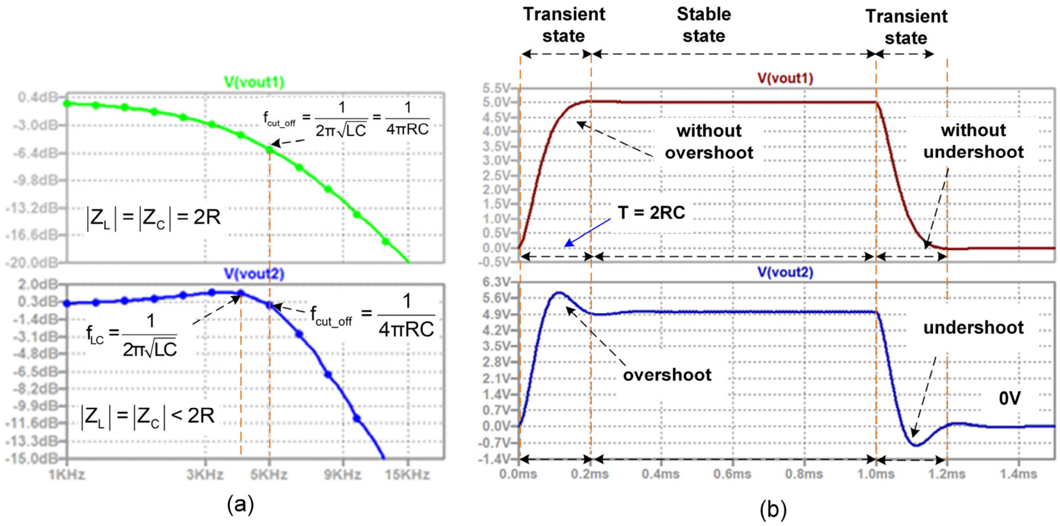


Fig.5. Frequency and time responses of RLC network, (a) Bode plot (frequency domain), (b) Step transient plot (time domain).

3.3 Time Behavior of System

It follows from Eq. 10 and Eq. 16 that the saturation time is defined by the time $\tau = 2RC$. After this time goes on, the output voltage is kept constant. In this case, the cut-off frequency is 5kHz, so the transient time of the step response is 0.2ms.

Let's define $s = j\omega$, and $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{2RC}$; Eq. 12 is rewritten as

$$H(s) = \frac{\frac{1}{LC}}{\left(j\omega + \frac{1}{2RC}\right)^2} = \frac{\omega^2}{(s+\omega)^2} \tag{20}$$

In the Laplace inversion transfer form, Eq. (20) is transferred as

$$h(t) = \mathrm{L}^{-1}\left\{\frac{\omega^2}{(s+\omega)^2}\right\} = \omega^2 te^{-\omega t} \tag{21}$$

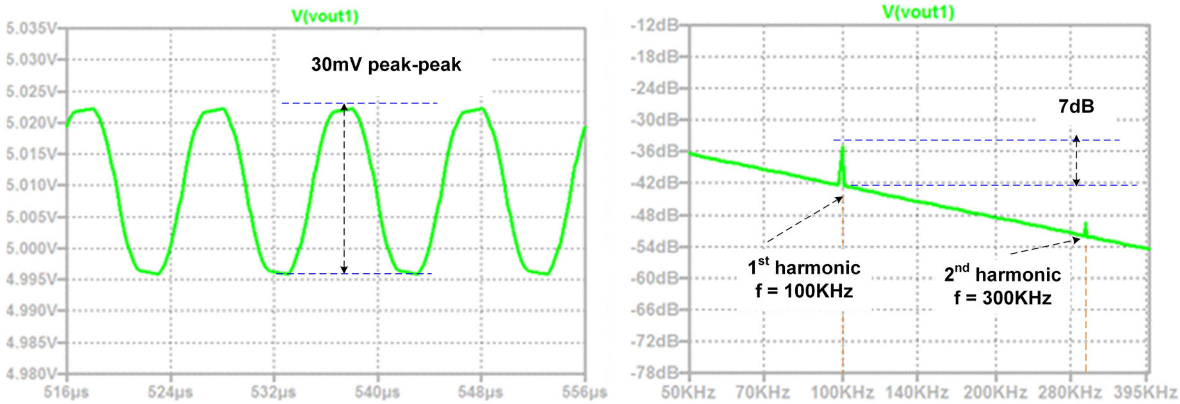
The output voltage is expressed by the switching input signals as

$$\frac{v_{out}(t)}{v_{in}(t)} = h(t) = \omega^2 te^{-\omega t} \Rightarrow v_{out}(t) = \omega^2 te^{-\omega t} v_{in}(t) \tag{22}$$

Let us rewrite Eq. (22) which includes the frequency variable

$$V_{out}(t) = \left(\frac{1}{2RC}\right)^2 te^{-\left(\frac{1}{2RC}\right)t} V_{in}(t) \tag{23}$$

In this model, the cutoff frequency is designed at $f_{cut\_off} = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{4\pi RC} = 5kHz$, and the fundamental frequency of the switch control is 100kHz. The waveform and spectrum of the output voltage at the steady state are shown in Fig. 6.



(a) (b)

Fig.6. Waveform of output voltage, (a) Steady-state ripples, (b) Spectrum noises.

The fundamental harmonic noise level is 7dB at 100kHz, and the second harmonic is 2dB at 300kHz. The output ripple level is 30mV peak to peak at the desired set voltage (5V). When the output ripple level is too high (30mVpp), this switching power converter is not acceptable in some applications.

3.4 EMI Harmonic Filter

The ripple noises consist of those at the fundamental frequency and the multiplied frequencies of these harmonics. Therefore, two notch harmonic filters are used to reduce these noises as shown in Fig. 7.

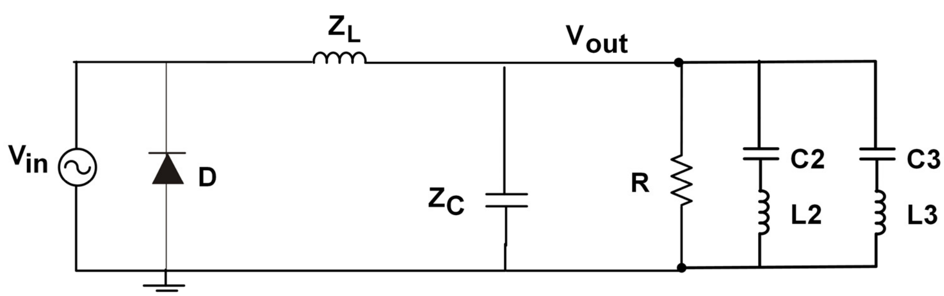


Fig.7. Notch harmonic filter in switching converter.

Let us employ the superposition principle at the set voltage node

$$V_{out}\left(\frac{1}{Z_L}+\frac{1}{Z_C}+\frac{1}{R}+\frac{1}{Z_{C2}+Z_{L2}}+\frac{1}{Z_{C3}+Z_{L3}}\right)=\frac{V_{in}}{Z_L} \tag{24}$$

The output voltage is reorganized as

$$V_{out}=V_{in}\frac{Z_C R\left(Z_{C2}+Z_{L2}\right)\left(Z_{C3}+Z_{L3}\right)}{\left\{\left[R\left(Z_C+Z_L\right)+Z_C Z_L\right]\left(Z_{C2}+Z_{L2}\right)+RZ_C Z_L\right\}\left(Z_{C3}+Z_{L3}\right)+RZ_C Z_L\left(Z_{C2}+Z_{L2}\right)} \tag{25}$$

Then, the transfer function of this system is written as

$$H = \frac{V_o}{V_{in}} = \frac{Z_C R\left(Z_{C2} + Z_{L2}\right)\left(Z_{C3} + Z_{L3}\right)}{\left\{\left[R\left(Z_C + Z_L\right) + Z_C Z_L\right]\left(Z_{C2} + Z_{L2}\right) + R Z_C Z_L\right\}\left(Z_{C3} + Z_{L3}\right) + R Z_C Z_L\left(Z_{C2} + Z_{L2}\right)} \quad (26)$$

The simplified transfer function of this system is expressed as

$$H(j\omega) = \frac{\frac{1}{LC}\left(\frac{1}{L_2C_2} + (j\omega)^2\right)\left(\frac{1}{L_3C_3} + (j\omega)^2\right)}{\left\{\left[(j\omega)^2 + \frac{1}{RC}(j\omega) + \frac{1}{LC}\right]\left(\frac{1}{L_2C_2} + (j\omega)^2\right) + \frac{1}{LC}\frac{(j\omega)}{L_2}\right\}\left(\frac{1}{L_3C_3} + (j\omega)^2\right) + \frac{1}{LC}\frac{(j\omega)}{L_3}\left(\frac{1}{L_2C_2} + (j\omega)^2\right)} \quad (27)$$

Here, $f_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi\sqrt{L_2C_2}}; f_3 = \frac{\omega_3}{2\pi} = \frac{1}{2\pi\sqrt{L_3C_3}};$ are the frequencies of the first and second harmonics at $f_{1^{st}\,harmonic} = \frac{1}{2\pi\sqrt{L_2C_2}} = 100kHz$, $f_{2^{nd}\,harmonic} = \frac{1}{2\pi\sqrt{L_3C_3}} = 300kHz$ (L2 = 3.18µH, C2 = 796nF, L3 = 531nH, C3 = 531nF). The selections of C2 and C3 will affect the charge and discharge times of the capacitor C. So, the values of C2 and C3 are very smaller than C. The frequency responses of this network are plotted in Fig. 8.

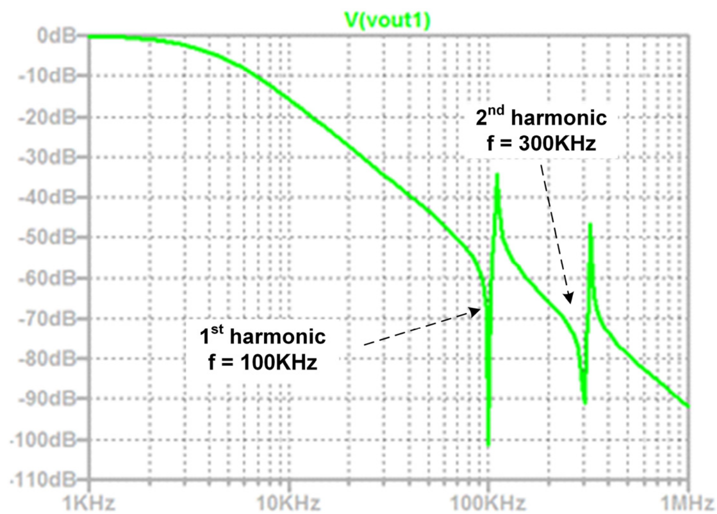


Fig.8. Frequency response of notch harmonic filter in switching converter.

3.5 Digital Charge Time Control of Network

A digital charge time control signal is used to operate a negative feedback loop. A simplified architecture of the feedback loop is shown in Fig. 9.

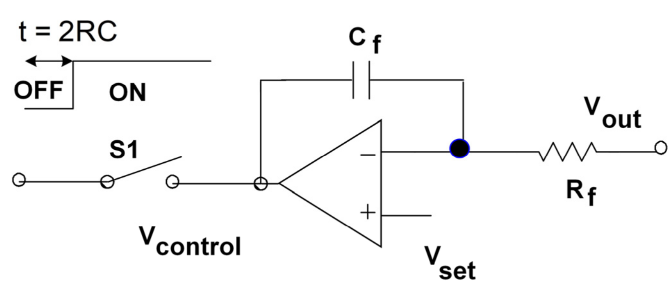


Fig.9. Negative feedback energy loop in switching power system.

This loop is controlled by a switch which is closed after the steady state. This switch turns ON the feedback loop the steady state $(t \geq 2RC)$. Employ the superposition principle at the set voltage node

$$V_{set}\left(\frac{1}{Z_f}+\frac{1}{R_f}\right)=\frac{V_{control}}{Z_f}+\frac{V_{out}}{R_f} \tag{28}$$

After simplifying Eq. (28), the control voltage is expressed as

$$V_{control}=V_{set}+\frac{Z_{Cf}}{R_f}\left(V_{set}-V_{out}\right)=V_{set}+\frac{1}{j\omega C_f R_f}\left(V_{set}-V_{out}\right) \tag{29}$$

It follows from Eq. (29) that the control voltage is dependent on the frequency (or duty cycle) of the control signal.

3.5 Design of Step-down Switching Power Converter

On this design as being described in Fig. 10, the fundamental and second harmonics of the control signals are rejected. The operation parameters of this design are described in Table 1.

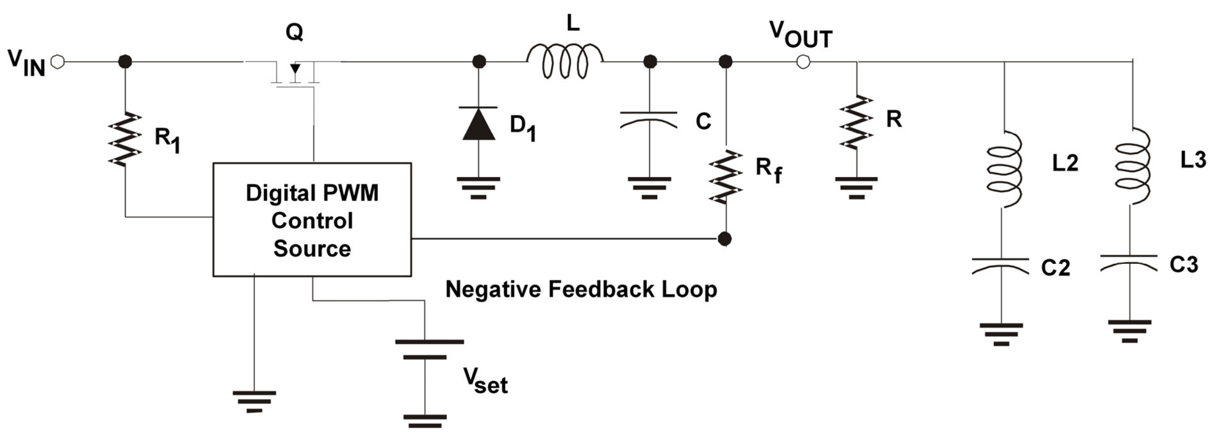


Fig.10. Step-down switching power conversion with notch harmonic filter.

Table 1: Operation parameters of step-down switching power conversion.

Input Voltage (Vin)	12V
Output Voltage (Vo)	5.0V
Output Current (Io)	1A
Clock Frequency (Fck)	100kHz
Current Step (⊿Io)	1A
Output Ripple	0.4mVpp
Over-shoot	0.1mV
Under-shoot	0.1mV

The transient time response of the set voltage is fast 0.2ms. The ripples of the output energy are largely reduced (smaller than 0.4mVpp) as shown in Fig. 11.

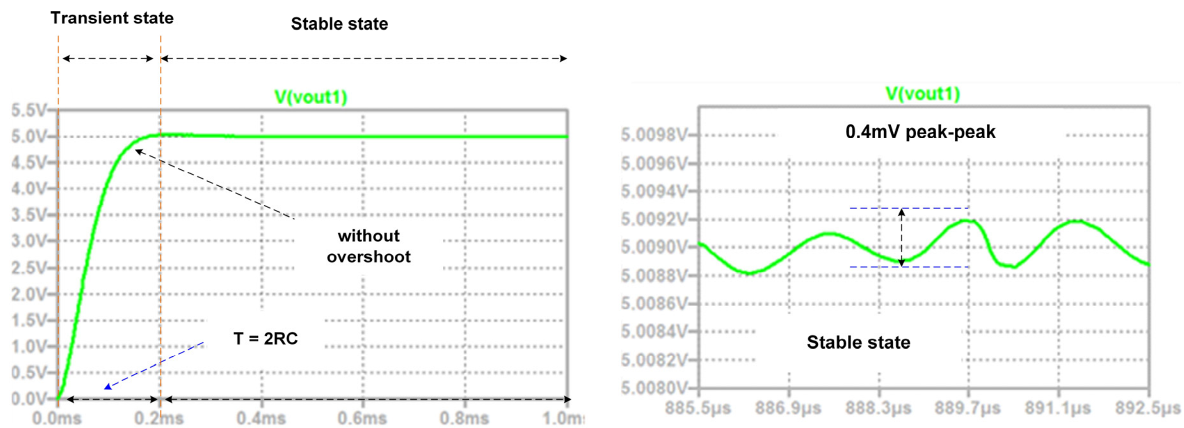


Fig.11. Transient responses and ripple levels of designed switching power converter.

4. Conclusion

A design of a switching power converter is analyzed and designed based on the superposition principle, and the modern optimal adaptive control theories. The overshoot and undershoot phenomena are perfectly controlled when the values of R, L and C are chosen by $|Z_L| = |Z_C| = 2R = 10\Omega$ .The cutoff frequency is designed as $f_{cut\_off} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{4\pi RC} = 5kHz$ (R = 5Ω, L = 318μH, C = 3.18μF). On the stable state 2ms$(t \geq 2RC)$, a digital time charge control signal is generated by a digital signal processing controller to control a voltage negative feedback loop. The EMI noises are reduced by two notch harmonic filters at $f_{1^{st}\,harmonic} = \frac{1}{2\pi\sqrt{L_2C_2}} = 100kHz$ and $f_{2^{nd}\,harmonic} = \frac{1}{2\pi\sqrt{L_3C_3}} = 300kHz$ (L2 = 3.18μH, C2 = 796nF, L3 = 531nH, C3 = 531nF). As a result, a switching power conversion system operates a fast time transient response and high stable output voltage, where the ripple is smaller than 0.4mVpp (or this ripple level is reduced from 30mVpp into 0.4mVpp). The EMI noises are significantly rejected (7dB reduction at the 1st harmonic, 2dB reduction at the 2nd harmonic). Therefore, this design is suitable in many low energy propagation applications. In future work, the input signals and parasitic of the R, L, and C and other components in these systems will be analyzed and modeled.

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