Uncertainty Evaluation of the Mechanical Properties in Drop-Ball Test using Three Coordinate Systems

Hadi Nasbey[1, a], Akihiro Takita [1,b], Agus Setyo Budi[2, c], Mitra Djamal[3,d] and Yusaku Fujii[1,e]

[1]Division of Mechanical Science and Technology, Faculty of Science and Technology, Gunma University, 1-5-1 Tenjin-cho, Gunma, 376-8515, JAPAN

[2]State University of Jakarta, Jalan Rawamangun muka no.1, Jakarta, INDONESIA

[3]High Theoretical Energy Physics and Instrumentation, Department of Physics, Bandung Institute of Technology, Bandung, INDONESIA

[a]<hadinasbey@gmail.com>, [b]<takita@gmail.com>, [c]<abihuda123@yahoo.com>, [d]<mitra@fi.itb.ac.id>, [e]<fujii@gunma-u.ac.jp>

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Abstract. In this study, the uncertainty of the mechanical properties in the Drop-ball test evaluated using three coordinate systems. In the drop-ball test, the spherical body containing a cube corner prism is dropped from an initial height and the inertial force acting on it is used as reference force. An optical interferometer is employed to obtain the velocity of the spherical body. Then the position and acceleration of the spherical body are calculated by integrating and differentiating the velocity, respectively. The force acting on the spherical body is calculated as the product of the mass and the acceleration. The triangle proportionality theorem is employed to calculate the mechanical properties in the measurement. The uncertainties of the mechanical properties are estimated using the general formula for error propagation.

1. Introduction

The force is an important quantity in the mechanical engineering field. The force is defined as the product of a mass and an acceleration. There are many methods can be conducted for measuring the force acting on the moving object such as using the strain gauge load cells, force transducers, and measuring force through pressure [1]. In our laboratory, we develop the Drop-ball test, which is a test to evaluate the dynamic force acting from the material under test when a spherical body slammed on it from an initial height. This test has been applied to various materials tests such as water, plastic and sand [2-4]. This test was modified from Levitation Mass Method (LMM), which successfully evaluated the dynamic force such as the impact response of a material, the microforce material tester, the electrical and mechanical responses of a force transducers, etc. [5-7]. In the LMM, the initial force of a rigid mass that levitated using a pneumatic linear bearing is used as the reference force. This force is applied to the object under test, such as a force transducer, a material or a structure. An optical interferometer is employed to measure the motion-induced time-varying beat frequency that produced by a moving of levitated mass. The aerostatic linear bearing is used to generate the negligible friction. The velocity, acceleration, and positions, are numerically calculated from the frequency based on the laws of physics.

In the drop-ball test, a spherical body is dropped from an initial height and collided with the test target. The total force acting on the spherical object is used as the reference force. This force acting on the spherical object is measured using an optical interferometer. The test has successfully evaluated impact force of a spherical body dropping onto a water surface [2], impact force of a plastic sheet [3], and sand particles [4]. In order to realize the 3D measurement of the mechanical properties in the drop-ball test, the theoretical calculation is proposed [8]. In the proposed method, the uncertainty of the 3D measurement of velocity is not yet evaluated.

In this paper, the general formula for error propagation is employed to estimate the uncertainty of the mechanical properties of the proposed method. This paper is improvements of the previous paper [9]. The improvements are a modification of the schematic diagram of the experimental setup of the 3D measurement of velocity in the Drop-ball test, estimation of the uncertainty of the force, and procedure to measure the angles between three light beams.

2. Experimental design

Fig. 1 shows the design of a schematic diagram of the experimental setup. The 3D measurement of velocity in the drop-ball test is conducted using three beams and interferometers. The Signal beam 1 (SB_1), b_1, is measured using the interferometer 1, which measure the velocity lie on the x-z plane. The Signal beam 2 (SB_2), b_2, is measured using the interferometer 2, which measure the velocity lie on the y-z plane. The Signal beam 3 (SB_3), b_3, is measured using the interferometer 3, which measure the velocity along the z-axis.

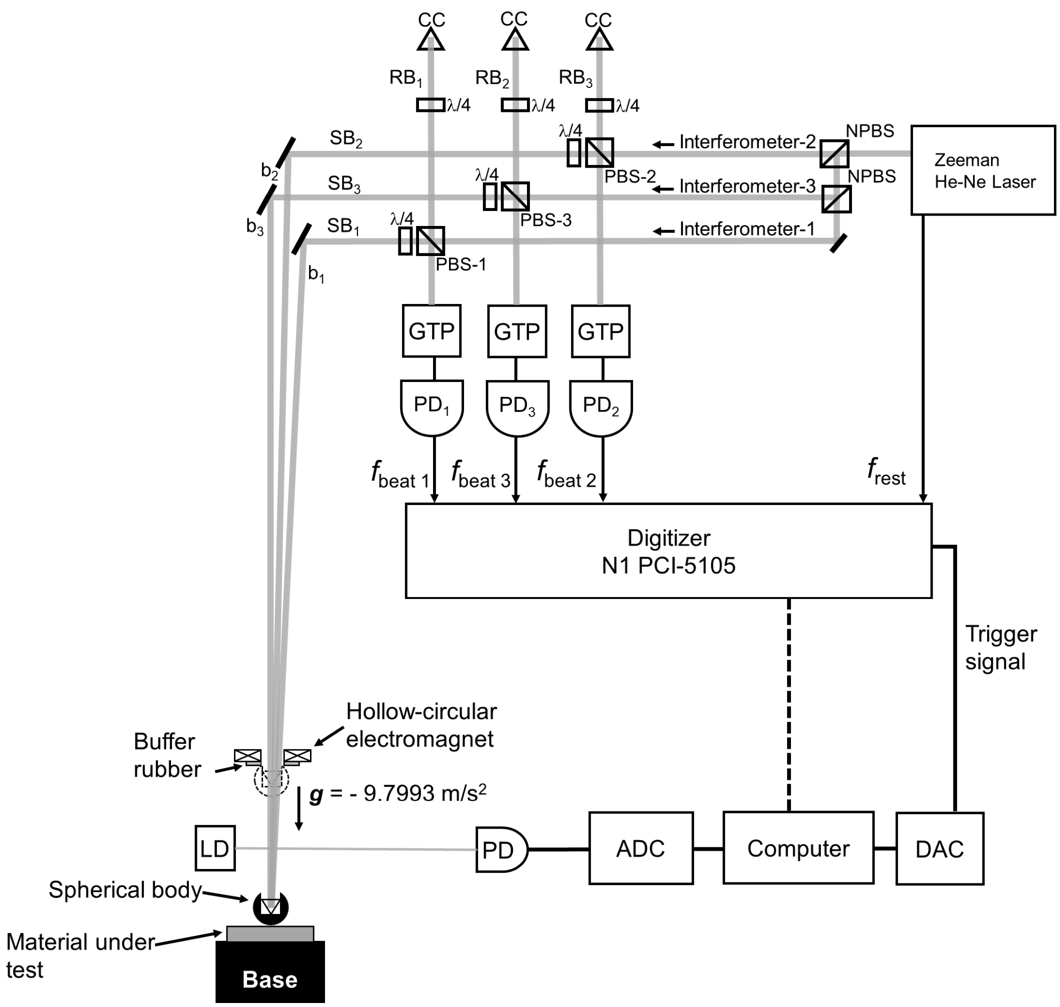

Fig. 1. Experimental setup for drop-ball test using three laser interferometers. (NPBS : Non-Polarizing beam splitter, PBS : Polarizing beam splitter, LD : Laser diode, PD : Photo diode, CC : Cube corner prism, GTP : Gland-Thomson prism, ADC : Analog to digital converter, DAC : Digital to analog converter, SB : Signal Beam, RBI: Reference Beam).

The spherical body is made of the SUS440 stainless steel with a diameter of approximately 30.2 mm. A cube corner prism, 12.7 mm in diameter, is attached inside the spherical body using an adhesive agent so that its optical center coincides with the center of gravity of the whole body. The total mass of the spherical body including the cube corner prism, $\boldsymbol{M}$, is approximately 93.88 g.

During the test, only the motion-induced time-varying beat frequency, $\boldsymbol{f}_{\mathbf{beat}}$, is measured and other quantities such as velocity, displacement, acceleration, and force, are calculated from the frequency. The velocity is calculated from the measured value of the Doppler shift frequency of the interferometer, $\boldsymbol{f}_{\mathbf{Doppler}}$, which expressed as follows:

$$\boldsymbol{v} = \lambda_{\mathbf{air}}\left(\boldsymbol{f}_{\mathbf{Doppler}}\right)/2 \tag{1}$$

$$\boldsymbol{f}_{\mathbf{Doppler}} = -\left(\boldsymbol{f}_{\mathbf{beat}} - \boldsymbol{f}_{\mathbf{rest}}\right) \tag{2}$$

Therefore, each velocity measured by each interferometer can be expressed as;

$$\boldsymbol{v}_{\mathbf{b1}} = -\lambda_{\mathbf{air}}\left(\boldsymbol{f}_{\mathbf{beat1}} - \boldsymbol{f}_{\mathbf{rest}}\right)/2 \tag{3}$$

$$\boldsymbol{v}_{\mathbf{b2}} = -\lambda_{\mathbf{air}}\left(\boldsymbol{f}_{\mathbf{beat2}} - \boldsymbol{f}_{\mathbf{rest}}\right)/2 \tag{4}$$

$$\boldsymbol{v}_{\mathbf{b3}} = -\lambda_{\mathbf{air}}\left(\boldsymbol{f}_{\mathbf{beat3}} - \boldsymbol{f}_{\mathbf{rest}}\right)/2\,, \tag{5}$$

where $\lambda_{\mathbf{air}}$ is the wavelength of the signal beam. $\boldsymbol{f}_{\mathbf{beat1}}$, $\boldsymbol{f}_{\mathbf{beat2}}$, and $\boldsymbol{f}_{\mathbf{beat3}}$, are the beat frequency from interferometer 1, 2, and 3, respectively. $\boldsymbol{f}_{\mathbf{rest}}$ is the rest frequency. $\boldsymbol{v}_{\mathbf{b1}}$, $\boldsymbol{v}_{\mathbf{b2}}$, $\boldsymbol{v}_{\mathbf{b3}}$, are velocity measured by interferometer 1, 2, and 3, respectively.

The beat frequency, $\boldsymbol{f}_{\mathbf{beat}}$, is a difference between the signal beam and the reference beam. The rest frequency, $\boldsymbol{f}_{\mathbf{rest}}$, is the frequency when the spherical body is at rest position and no Doppler shift is added to the signal beams. The force acting on the spherical body is calculated as the multiplication of the mass and the acceleration of the spherical body. The acceleration of the spherical body is calculated by differentiating the velocity. The displacement is calculated by integrating the velocity.

The total force acting on the spherical body, $\boldsymbol{F}_{\mathbf{mass}}$, is a sum of the weight of the spherical body and the impact force acting from the material under test when it slammed by the spherical body, $\boldsymbol{F}_{\mathbf{impact}}$, with a condition the other forces such as the air drag and the magnetic force from hollow-circular electromagnet are negligible. The total force acting on the mass can be express as:

$$\boldsymbol{F}_{\mathbf{mass}} = -\boldsymbol{Mg} + \boldsymbol{F}_{\mathbf{impact}} \tag{6}$$

Then the impact force acting from the material under test can be expressed as:

$$\boldsymbol{F}_{\mathbf{impact}} = \boldsymbol{F}_{\mathbf{mass}} + \boldsymbol{Mg} \tag{7}$$

A Zeeman-type two-wavelength He-Ne laser is used as the light source of the interferometers, which have two frequencies with orthogonal polarization. The interference signals are detected using photodetector (PD_1, PD_2, and PD_3) and the output signals are recorded using the Digitizer (NI PCI-5105, National Instruments Corp., USA). The Zero-Crossing Fitting method (ZFM) [10] is employed to obtain the beat frequencies, $\boldsymbol{f}_{\mathbf{beat\,1}}$, $\boldsymbol{f}_{\mathbf{beat\,2}}$, $\boldsymbol{f}_{\mathbf{beat\,3}}$, and rest frequency, $\boldsymbol{f}_{\mathbf{rest}}$.

3. Result

The velocity of the spherical body at the optical center of the cube corner prism, which is made to coincide with the center of gravity of the spherical body with the uncertainty of 0.2 mm, are measured with three light beams, b_1, b_2, b_3. The measurement region in this method is defined as a range in which the interference fringes are detected by three interferometers simultaneously.

In the initial stage, three light beams are arranged following the Fig.2. Where P_1, P_2, and P_3 are mirrors. α is an angle between b_1 and b_3 and β is an angle between b_2 and b_3. O is the crossing point between the three light beams.

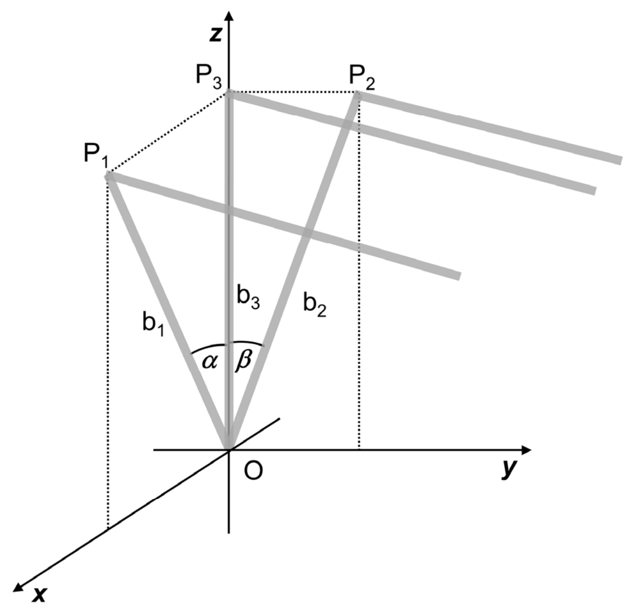

Fig. 2. The diagram beam of laser reflected from mirror to point of measurement.

The velocity of the spherical body along the measurement region is expressed as the linear combination of three unit vectors;

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \quad (8)$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (9)$$

where $\vec{v}$ is the vector of velocity, v_x, v_y, v_z, are scalar quantities of the vectors of velocity along x-axis, y-axis, and z-axis, respectively. $\hat{i}$, $\hat{j}$, $\hat{k}$, are unit the vectors along x-axis, y-axis, and z-axis, respectively.

3.1 Calculation of the velocity along z-axis, $\vec{v}_z$

The interferometer 3 is used to measure the velocity along z-axis, $\vec{v}_z$. Beam 3, b_3, is adjusted to coincide to the z-axis. The velocity along z-axis, $\vec{v}_z$, can be expressed as:

$$\vec{v}_z = v_z\hat{k} \quad (10)$$

$$\boldsymbol{v}_{\mathrm{z}} = \boldsymbol{v}_{\mathrm{b3}} \tag{11}$$

3.2 Calculation of the velocity along x-axis, $\vec{v}_{\mathrm{x}}$

The interferometer 1 is used to measure the velocity along x-z plane, $\boldsymbol{v}_{\mathrm{b1}}$, and the interferometer 3 is used to measure the velocity along z-axis, $\boldsymbol{v}_{\mathrm{z}}$. The triangle proportionality theorem is employed to obtain the velocity along x-axis, $\boldsymbol{v}_{\mathrm{x}}$ (Fig.3). It expressed as:

$$\boldsymbol{v}_{\mathrm{b1}} = \boldsymbol{v}_{\mathrm{x}} \sin\boldsymbol{\alpha} + \boldsymbol{v}_{\mathrm{b3}} \cos\boldsymbol{\alpha} \tag{12}$$

$$\boldsymbol{v}_{\mathrm{x}} = \frac{\boldsymbol{v}_{\mathrm{b1}} - \boldsymbol{v}_{\mathrm{b3}} \cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}} \tag{13}$$

$$\vec{v}_{\mathrm{x}} = \boldsymbol{v}_{\mathrm{x}} \hat{\boldsymbol{i}} \tag{14}$$

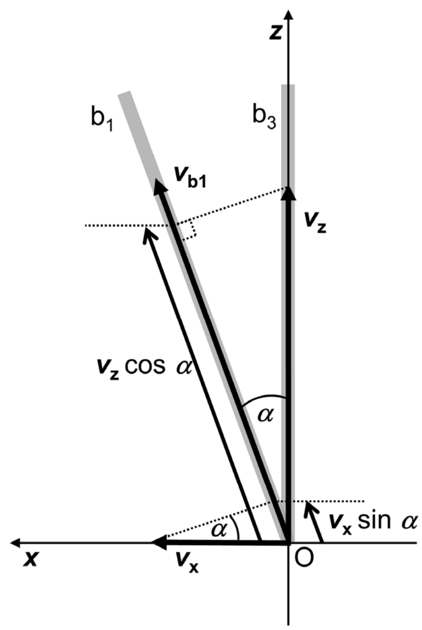

Fig. 3. The diagram of the concept measuring velocity of the dropping ball with two beam of laser which at z-axis and x-z plane.

3.3 Calculation of the velocity along y-axis, $\vec{v}_{\mathrm{y}}$

The interferometer 2 is used to measure the velocity along y-z plane. The velocity along y-axis, $\boldsymbol{v}_{\mathrm{y}}$, is calculated using similar way as calculating $\boldsymbol{v}_{\mathrm{x}}$. $\vec{v}_{\mathrm{y}}$ is expressed as:

$$\boldsymbol{v}_{\mathrm{b2}} = \boldsymbol{v}_{\mathrm{y}} \sin\boldsymbol{\beta} + \boldsymbol{v}_{\mathrm{b3}} \cos\boldsymbol{\beta} \tag{15}$$

$$\boldsymbol{v}_{\mathrm{y}} = \frac{\boldsymbol{v}_{\mathrm{b2}} - \boldsymbol{v}_{\mathrm{b3}} \cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}} \tag{16}$$

$$\vec{v}_{\mathrm{y}} = \boldsymbol{v}_{\mathrm{y}} \hat{j} \tag{17}$$

The velocity of the spherical body along the measurement region is calculated by substitute Eq. (11), Eq. (13), Eq. (16) to Eq. (10). Then the velocity can be expressed as:

$$\vec{v} = \left(\frac{\boldsymbol{v}_{\mathbf{b1}} - \boldsymbol{v}_{\mathbf{b3}} \cos \boldsymbol{\alpha}}{\sin \boldsymbol{\alpha}} \right) \hat{\boldsymbol{i}} + \left(\frac{\boldsymbol{v}_{\mathbf{b2}} - \boldsymbol{v}_{\mathbf{b3}} \cos \boldsymbol{\beta}}{\sin \boldsymbol{\beta}} \right) \hat{\boldsymbol{j}} + \left(\boldsymbol{v}_{\mathbf{b3}} \right) \hat{\boldsymbol{k}} \tag{18}$$

$$\boldsymbol{v} = \sqrt{\left(\frac{\boldsymbol{v}_{\mathbf{b1}} - \boldsymbol{v}_{\mathbf{b3}} \cos \boldsymbol{\alpha}}{\sin \boldsymbol{\alpha}} \right)^2 + \left(\frac{\boldsymbol{v}_{\mathbf{b2}} - \boldsymbol{v}_{\mathbf{b3}} \cos \boldsymbol{\beta}}{\sin \boldsymbol{\beta}} \right)^2 + \left(\boldsymbol{v}_{\mathbf{b3}} \right)^2} \tag{19}$$

The acceleration is calculated from differentiating the velocity and the force is calculated as the mass multiplied by the acceleration. The acceleration and the force are expressed as:

$$\boldsymbol{a} = \sqrt{\left(\frac{\boldsymbol{a}_{\mathbf{b1}} - \boldsymbol{a}_{\mathbf{b3}} \cos \boldsymbol{\alpha}}{\sin \boldsymbol{\alpha}} \right)^2 + \left(\frac{\boldsymbol{a}_{\mathbf{b2}} - \boldsymbol{a}_{\mathbf{b3}} \cos \boldsymbol{\beta}}{\sin \boldsymbol{\beta}} \right)^2 + \left(\boldsymbol{a}_{\mathbf{b3}} \right)^2} \tag{20}$$

$$\boldsymbol{F} = \boldsymbol{M}\boldsymbol{a} \tag{21}$$

3.4 Measurement of α and β

The displacement is calculated by integrating Eq. (12) and Eq. (15). The displacement can be expressed as

$$\boldsymbol{l}_{\mathbf{b1}} = \boldsymbol{l}_x \sin \boldsymbol{\alpha} + \boldsymbol{l}_{\mathbf{b3}} \cos \boldsymbol{\alpha} \tag{22}$$

$$\boldsymbol{l}_{\mathbf{b2}} = \boldsymbol{l}_y \sin \boldsymbol{\beta} + \boldsymbol{l}_{\mathbf{b3}} \cos \boldsymbol{\beta}, \tag{23}$$

where $\boldsymbol{l}_{b1}$ is the displacement measured by interferometer 1, $\boldsymbol{l}_{b2}$ is the displacement measured by interferometer 2, $\boldsymbol{l}_{b3}$, is the displacement measured by interferometer 3, $\boldsymbol{l}_x$ and $\boldsymbol{l}_y$ are the displacement along x-axis and y-axis, respectively, which are measured using a micrometer of multi-axis translation stage. The angle between b_1 and b_3, $\boldsymbol{\alpha}$, is measured using the solution of Eq. (22), which expressed as

$$\boldsymbol{\alpha} = \sin^{-1} \frac{\left(\boldsymbol{l}'_{\mathbf{b3}} \boldsymbol{l}_{\mathbf{b1}} - \boldsymbol{l}'_{\mathbf{b1}} \boldsymbol{l}_{\mathbf{b3}} \right)}{\left(\boldsymbol{l}'_{\mathbf{b3}} \boldsymbol{l}_x - \boldsymbol{l}'_x \boldsymbol{l}_{\mathbf{b3}} \right)}, \tag{24}$$

where $\boldsymbol{l}_{\mathbf{b1}}, \boldsymbol{l}_{\mathbf{b3}}, \boldsymbol{l}_x$, are the displacements of fist measurement and $\boldsymbol{l}'_{\mathbf{b1}}, \boldsymbol{l}'_{\mathbf{b3}}, \boldsymbol{l}'_x$, are the displacements of second measurement, during the process of measuring $\boldsymbol{\alpha}$.

The following steps are conducted to measure the angles ($\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$);

1. The multi-axis translation stage with x-y-z configuration is employed to measure the angles ($\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$).
2. The cube corner prism is fixed at the center of the multi-axis translation stage table.
3. Three light beams, b_1, b_2, b_3, are set up based on Fig. 2.
4. The cube corner prism on the center of the multi-axis translation stage table is placed at the crossing point between the three light beams, O, so the interference fringes are detected by all three of interferometers.

5. For measuring $\boldsymbol{\alpha}$, the cube corner prism is moved along the x-axis and z-axis simultaneously by a certain length, $\boldsymbol{l}_x$, and $\boldsymbol{l}_z$, which does not exceed the measurement region. This is the first movement for measuring $\boldsymbol{\alpha}$.
6. During the movement along the x-axis, the movement length, $\boldsymbol{l}_{b1}$, $\boldsymbol{l}_{b3}$, are measured by the interferometers.
7. In the second movement, the cube corner prism is moved along the along the x-axis and z-axis simultaneously by a certain length, $\boldsymbol{l}'_x$, $\boldsymbol{l}'_z$, which does not exceed the measurement region. The $\boldsymbol{l}_z$ must not equal to $\boldsymbol{l}'_z$ ($\boldsymbol{l}_z \neq \boldsymbol{l}'_z$).
8. During the movement along the x-axis, the movement length, $\boldsymbol{l}'_{\mathbf{b1}}$, $\boldsymbol{l}'_{\mathbf{b3}}$, are measured by the interferometers.
9. The angle between b_1 and b_3, $\boldsymbol{\alpha}$, is calculated by Eq. (24).
10. The angle between b_2 and b_3, $\boldsymbol{\beta}$, is measured using the similar way as measuring the $\boldsymbol{\alpha}$ using the following equation;

$$\boldsymbol{\beta} = \sin^{-1}\frac{\left(\boldsymbol{l}'_{\mathbf{b3}}\boldsymbol{l}_{\mathbf{b2}} - \boldsymbol{l}'_{\mathbf{b2}}\boldsymbol{l}_{\mathbf{b3}}\right)}{\left(\boldsymbol{l}'_{\mathbf{b3}}\boldsymbol{l}_y - \boldsymbol{l}'_y\boldsymbol{l}_{\mathbf{b3}}\right)}, \tag{25}$$

where $\boldsymbol{l}_{\mathbf{b2}}$, $\boldsymbol{l}_{\mathbf{b3}}$, $\boldsymbol{l}_y$, are the displacements of fist measurement and $\boldsymbol{l}'_{\mathbf{b2}}$ $\boldsymbol{l}'_{\mathbf{b3}}$ $\boldsymbol{l}'_y$ are the displacements of second measurement, during the process of measuring $\boldsymbol{\beta}$.

3.5 Calculation of the uncertainty of the acceleration

The acceleration of the spherical body is calculated by differentiating the velocity. The scalar of the acceleration vector is defined with Eq. (20). The general formula for error propagation is employed to calculate the uncertainty of the acceleration. The uncertainty of the acceleration of the spherical body can be express as

$$\Delta\boldsymbol{a} = \sqrt{\left(\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{\alpha}}\Delta\boldsymbol{\alpha}\right)^2 + \left(\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{\beta}}\Delta\boldsymbol{\beta}\right)^2 + \left(\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{a}_{\mathrm{b1}}}\Delta\boldsymbol{a}_{\mathrm{b1}}\right)^2 + \left(\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{a}_{\mathrm{b2}}}\Delta\boldsymbol{a}_{\mathrm{b2}}\right)^2 + \left(\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{a}_{\mathrm{b3}}}\Delta\boldsymbol{a}_{\mathrm{b3}}\right)^2}, \tag{26}$$

where;

$$\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{\alpha}} = \frac{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)\left(\dfrac{\boldsymbol{a}_{\mathbf{b3}} - \boldsymbol{a}_{\mathbf{b1}}\cos\boldsymbol{\alpha}}{\sin^2\boldsymbol{\alpha}}\right)}{\sqrt{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)^2 + \left(\dfrac{\boldsymbol{a}_{\mathbf{b2}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)^2 + \left(\boldsymbol{a}_{\mathbf{b3}}\right)^2}} \tag{27}$$

$$\frac{\partial\boldsymbol{a}}{\partial\boldsymbol{\beta}} = \frac{\left(\dfrac{\boldsymbol{a}_{\mathbf{b2}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)\left(\dfrac{\boldsymbol{a}_{\mathbf{b3}} - \boldsymbol{a}_{\mathbf{b2}}\cos\boldsymbol{\beta}}{\sin^2\boldsymbol{\beta}}\right)}{\sqrt{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)^2 + \left(\dfrac{\boldsymbol{a}_{\mathbf{b2}} - \boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)^2 + \left(\boldsymbol{a}_{\mathbf{b3}}\right)^2}} \tag{28}$$

$$\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{a}_{\mathbf{b1}}}=\frac{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)\left(\dfrac{1}{\sin\boldsymbol{\alpha}}\right)}{\sqrt{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)^2+\left(\dfrac{\boldsymbol{a}_{\mathbf{b2}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)^2+\left(\boldsymbol{a}_{\mathbf{b3}}\right)^2}} \tag{29}$$

$$\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{a}_{\mathbf{b2}}}=\frac{\left(\dfrac{\boldsymbol{a}_{\mathbf{b2}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)\left(\dfrac{1}{\sin\boldsymbol{\beta}}\right)}{\sqrt{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)^2+\left(\dfrac{\boldsymbol{a}_{\mathbf{b2}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)^2+\left(\boldsymbol{a}_{\mathbf{b3}}\right)^2}} \tag{30}$$

$$\frac{\partial \boldsymbol{a}}{\partial \boldsymbol{a}_{\mathbf{b3}}}=\frac{\left(\dfrac{\left(\boldsymbol{a}_{\mathbf{b3}}-\boldsymbol{a}_{\mathbf{b1}}\right)\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)+\left(\dfrac{\left(\boldsymbol{a}_{\mathbf{b3}}-\boldsymbol{a}_{\mathbf{b2}}\right)\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)+\boldsymbol{a}_{\mathbf{b3}}}{\sqrt{\left(\dfrac{\boldsymbol{a}_{\mathbf{b1}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\alpha}}{\sin\boldsymbol{\alpha}}\right)^2+\left(\dfrac{\boldsymbol{a}_{\mathbf{b2}}-\boldsymbol{a}_{\mathbf{b3}}\cos\boldsymbol{\beta}}{\sin\boldsymbol{\beta}}\right)^2+\left(\boldsymbol{a}_{\mathbf{b3}}\right)^2}} \tag{31}$$

$\Delta\boldsymbol{a}$ is the uncertainty of the acceleration of the spherical body, $\Delta\boldsymbol{a}_{\mathbf{b1}}$ is the root mean square values of the standard deviation of the acceleration before the spherical body collided with the materials under test, which measure on the xz plane, $\Delta\boldsymbol{a}_{\mathbf{b2}}$ is the root mean square values of the standard deviation of the acceleration before the spherical body collided with the materials under test, which measure on the yz plane, $\Delta\boldsymbol{a}_{\mathbf{b3}}$ is the root mean square values of the standard deviation of the acceleration before the spherical body collided with the materials under test, which measure along z-axis, $\Delta\boldsymbol{\alpha}$ is the uncertainty of $\boldsymbol{\alpha}$ and $\Delta\boldsymbol{\beta}$ is the uncertainty of $\boldsymbol{\beta}$.

3.6 Calculation of the uncertainty of the force

The force acting on the spherical body is calculated as product of mass and acceleration, which express in Eq. (21). The mass of the spherical body is measured using the electrical balance with a standard uncertainty of approximately 0.01 g. The acceleration of the spherical body is estimated using the Eq. (26). Then the uncertainty of the force acting on the spherical body can be expressed as

$$\Delta\boldsymbol{F}=\sqrt{\left(\boldsymbol{a}\times\Delta\boldsymbol{M}\right)^2+\left(\boldsymbol{M}\times\Delta\boldsymbol{a}\right)^2} \tag{32}$$

where $\Delta\boldsymbol{F}$ is the uncertainty of the force acting on the spherical body, $\Delta\boldsymbol{M}$ is uncertainty of the mass of the spherical body, and $\Delta\boldsymbol{a}$ is the uncertainty of the acceleration of the spherical body.

4. Discussion

This study focuses on calculating dynamic force as the product of mass and acceleration. The acceleration is calculated by differentiating the velocity and the uncertainty of the acceleration is estimated using the general formula for error propagation. The uncertainty of the other quantities such as displacement and velocity can be calculated using the same way as the uncertainty of the acceleration.

In the proposed method only time-varying beat frequency is measured using an interferometer. The velocity is calculated from the time-varying beat frequency. The displacement and acceleration are calculated from the velocity. The force is calculated as the product of mass and acceleration. Those result a good synchronization between the quantities. The angles of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are measured using the Eq. 24 and Eq. 25. The standard uncertainty of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are estimated 0.01 mm at 1 m which equal to 1 mrad.

5. Conclusion

The uncertainty of the mechanical properties in the Drop-ball Test using three coordinate systems were evaluated. Three interferometers are used to measure the velocity along the z-axis, lie on the x-z plane and lie on the y-z plane. The triangle proportionality theorem is employed to calculate the velocity along x-axis and y-axis. The velocity of the spherical body is expressed in the vector of velocity using three coordinate system, *i.e.* $\boldsymbol{v}_x$, $\boldsymbol{v}_y$, $\boldsymbol{v}_z$. The acceleration of the spherical body is calculated using the similar way to the velocity. Three value of the accelerations, $\boldsymbol{a}_{b1}, \boldsymbol{a}_{b2}, \boldsymbol{a}_{b3}$, are obtained by differentiating the velocities, $\boldsymbol{v}_{b1}, \boldsymbol{v}_{b2}, \boldsymbol{v}_{b3}$. Then the acceleration is formulated as the function of five variables ($\boldsymbol{a}_{b1}, \boldsymbol{a}_{b2}, \boldsymbol{a}_{b3}, \boldsymbol{\alpha}, \boldsymbol{\beta}$). The uncertainty of the acceleration is estimated using the general formula for error propagation. The dynamic force is calculated as the product of the mass and acceleration. The uncertainty of the dynamic force is estimated.

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