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Expansion to parameter estimation problem of modified Kalman filter with stochastic dynamic analysis in prediction procedure

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Abstract. The modified Kalman Filter incorporating Stochastic Dynamic Analysis in prediction step (KF-SDA) was already introduced in previous paper for systematically account for the nonlinear and parametric uncertainty excitation systems. In conventional study, the fundamental verifications were performed using single degree-of-freedom (DOF) system, two-DOF system, single-DOF system which is simultaneously subjected to stochastic parametric excitations and nonlinear single-DOF system, respectively. On the other hand, the actual field data analysis is often desired the methodology of the unknown parameter estimation using observable sensor data, for example, online anomaly detection, diagnosis, and so on. However, the conventional KF-SDA was not conducted the expansion to parameter estimation problem. This paper deals with the expansion method of KF-SDA to a parameter estimation problem. The parameter estimation problem based on KF-SDA will derived using the new state space representation. The fundamental operation verification is performed in the case of single-DOF system which is subjected to white noise excitation. Furthermore, application to the nonlinear system which is subjected to white noise excitation and the random parametric excitation system are considered.

1. Introduction

Filtering is important in various industrial applications for efficient operation using actual field data. Filters were first introduced by Norbert Wiener in the 1940s. Twenty years later, Kalman and Bucy had successfully demonstrated the use of filters in digital signal processing [1,2]. In particular, they showed that modelling uncertainty is important for state estimations [3,4,5].

Extensive research has been conducted on Kalman filters which consider modeling uncertainty [3,4,5]. In a Kalman filter, the system parameters are modeled by random perturbations based on a stochastic model with time-dependent noise (or multiplicative noise). Previously, the Kalman-type filtering problem for a linear system with parametric uncertainties was considered [5,6,7]. In those previous studies, a benchmark test with the classical Kalman filter was also introduced; as a result, the accuracy of the above-mentioned previous method was higher, compared with methods that use classical filtering approaches. However, the above state estimator cannot be used on nonlinear systems, because it assumes a linear Gaussian process. Thus, an approach that systematically deals with nonlinear systems and parametric uncertainty excitation systems is required.

In light of the above, in a previous paper we proposed a modified Kalman filter that incorporates stochastic dynamic analysis (KF-SDA) into its prediction process [8]. The proposed KF-SDA was derived by rewriting the formula for the prediction process in the classical Kalman filtering approach. The prediction process of the classical Kalman filter is composed of the update process, estimation of the state value, and estimation of the error covariance matrices. The update process obeys the dynamics of the Ito probabilistic differential equation of a linear system. Contrary to the previous Kalman filter, in our proposed method, in the prediction process we replace the Ito probabilistic differential equation by the moment equation.

In a previous paper [8], the fundamental operation of the proposed method was numerically validated for a system with a single degree of freedom (single-DOF system) and for a system with two degrees of freedom (two-DOF system), subjected to white noise. Furthermore, a single-DOF system subjected simultaneously to white noise and random parametric excitation was considered. In addition, the Duffing system subjected to white noise was also considered.

On the other hand, one of the typical applications of the Kalman filter is the well-studied parameter estimation problem [9,10,11]. Applications to frequency tracking [9], structural parameter estimation [10], and correspondence of modeling uncertainty [11] have been considered previously. Thus, our proposed Kalman filter was evaluated in the context of the parameter estimation problem.

In this paper, we described an expansion method for the parameter estimation problem of the proposed KF-SDA. The parameter estimation problem of KF-SDA is derived by considering that the parameters' dynamics are described by the Brownian motion. The fundamental operation is numerically validated for a single-DOF system subjected to white noise. Furthermore, applications to third-order (i.e., symmetrical restoring force) and second-order (i.e., asymmetrical restoring force) nonlinear systems subjected to white noise are considered. Moreover, an application to a single-DOF system in the case of a unit mass subjected simultaneously to random parametric and white noise excitations is considered.

2. Extension to the parameter estimation problem of KF-SDA

The parameter estimation problem of KF-SDA is formulated by assuming that the unknown parameters' dynamics are described by the Brownian motion. The state space representation of the parameter estimation problem is as follows:

State equation

$$\frac{d\boldsymbol{\theta}(t)}{dt} = \boldsymbol{\gamma} v(t) \tag{1}$$

Observation equation

$$y(t) = \boldsymbol{c}(t)\boldsymbol{\theta}(t) + w(t) \tag{2}$$

where, $\boldsymbol{\theta}$ is the $1 \times n$ vector of unknown parameters. The right-hand-side term $\boldsymbol{\gamma} v(t)$ in the state equation represents the white noise excitation. Thus, the unknown parameters' dynamics are described by the Brownian motion. The moment equation is as follows:

$$\frac{d\mathrm{E}[h]}{dt} = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{E}\left[\left(\frac{\partial^2 h}{\partial\theta_i \partial\theta_j}\right)[\boldsymbol{\gamma} D \boldsymbol{\gamma}^T]_{i,j}\right] \tag{3}$$

The moment equation describes the temporal evolution of the system. Here, the right-hand-side term represents the contribution from the second-order moment. $\mathrm{E}[\cdot]$ represents the expected value of the bracketed quantity, while h is the product of stochastic variables. For example, $h = \theta_1^i \theta_2^j \dots \theta_n^k$. The indices $i, j, \dots, k$ are constrained by the moment order. For example, in the case of the first-order moment, the constraint is $i + j + \cdots + k = 1$(i.e., the allowed index combinations are $i = 1, j = 0, \dots, k = 0$, $i = 0, j = 1, \dots, k = 0$, and $i = 0, j = 0, \dots, k = 1$). Moreover, D represents the diffusion coefficient of the system noise. Here, first-order moments are identified as unknown parameters, as a $\mathrm{E}[\theta_i] = \theta_i$ $(i = 1, \dots, n)$. Moreover, second-order moments are identified as a prior covariance matrix of unknown parameters, as follows:

$$P^{-} = \begin{bmatrix} \mathrm{E}[\theta_i^2] & \cdots & \mathrm{E}[\theta_i\theta_n] \\ \vdots & \ddots & \vdots \\ \mathrm{E}[\theta_n\theta_i] & \cdots & \mathrm{E}[\theta_n^2] \end{bmatrix} \tag{4}$$

The solution of the moment equation (i.e., the average vector and covariance matrix) is analytically obtained, as follows:

Solution of the first-order moment equation

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t-1) \tag{5}$$

Solution of the second-order moment equation

$$P(t) = P(t-1) + 2D\boldsymbol{\gamma}\boldsymbol{\gamma}^T dt \tag{6}$$

The, discrete time increment of dt denotes the sampling time. The proposed KF-SDA is obtained by replacing the formula for the prediction process using the above analytical solutions of the moment equation. The observation equation (Eq. (2)) is composed of the governing equation. The observation matrix $\boldsymbol{c}(t)$ contains instantaneous sensor data.

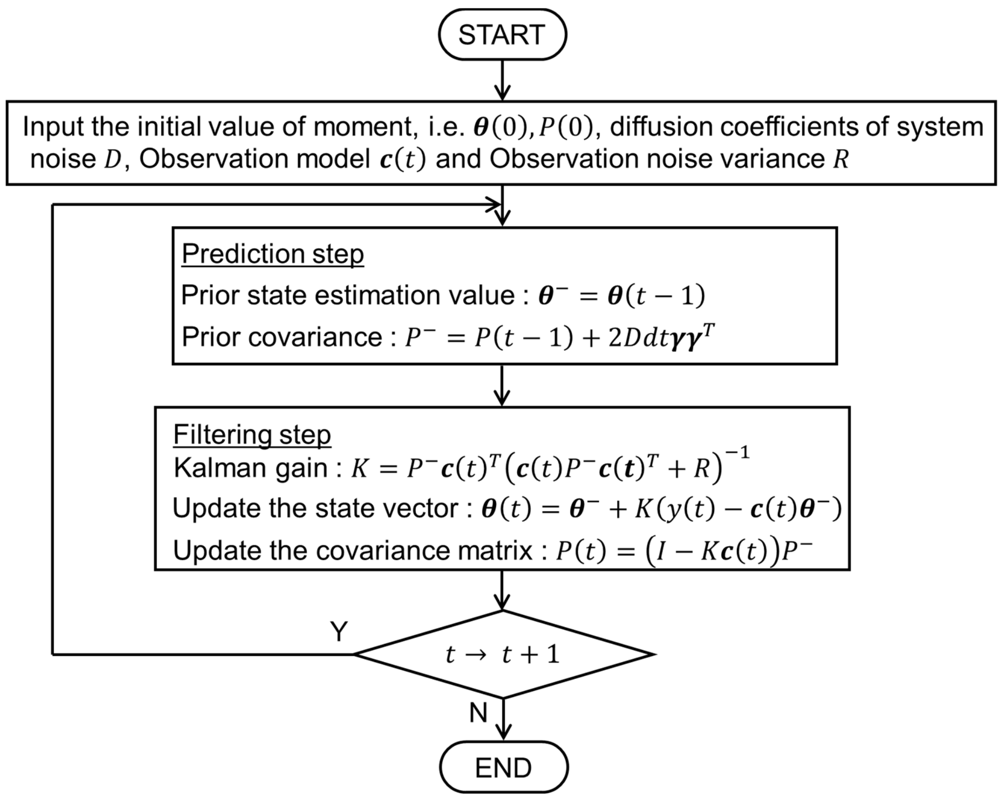


Fig. 1. Flowchart of the parameter estimation problem using the proposed Kalman filter (KF-SDA).

Based on the above considerations, the flowchart of the parameter estimation problem using KF-SDA is shown in Fig. 1. First, the proposed KF-SDA requires initial values in terms of the unknown parameters, the diffusion coefficient of the system noise, the observation model, and the variance of the observation noise. The state vector is defined based on the initial values of the unknown parameters. The prior state vector $\boldsymbol{\theta}^{-}$ and the prior covariance P^{-} are obtained by the analytical solution of the moment equation. Furthermore, in the filtering step, the Kalman gain is calculated as in the classical Kalman filtering approach. Here, the observation matrix $\boldsymbol{c}(t)$ is defined in terms of the sensor data. Thus, the observation matrix should be updated after each iteration step. The prior state estimated values are updated using the residuals between the observation time series $y(t)$ and the predicted values $\boldsymbol{c}(t)\boldsymbol{\theta}^{-}$. Moreover, the prior covariance is updated using the Kalman gain and the observation model. Parameter estimation based on the proposed KF-SDA is realized by alternating between the prediction and filtering steps.

3. Numerical simulation for validating the fundamental operation

Below, we describe numerical simulations of the parameter estimation using the proposed KF-SDA. The fundamental operation was validated using a linear single-DOF system excited by white noise.

3.1 Mathematical formulation

The governing equation of a single-DOF system for a unit mass subjected to white noise is as follows:

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = w, \tag{7}$$

where x denotes the displacement, c denotes the damping coefficient, k denotes the spring constant, and w denotes the white noise excitation force. Here, the signals are assumed to be displacement, velocity, and acceleration. The state-space representation of this single-DOF system for the parameter estimation problem is as follows:

State equation

$$\frac{d\boldsymbol{\theta}(t)}{dt} = \boldsymbol{\gamma}v(t), \boldsymbol{\theta}(t) = \begin{bmatrix} k(t) \\ c(t) \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{8}$$

Observation equation

$$\ddot{x}(t) = \boldsymbol{c}(t)\boldsymbol{\theta}(t) + w(t), \boldsymbol{c}(t) = [-x(t) \quad -\dot{x}(t)] \tag{9}$$

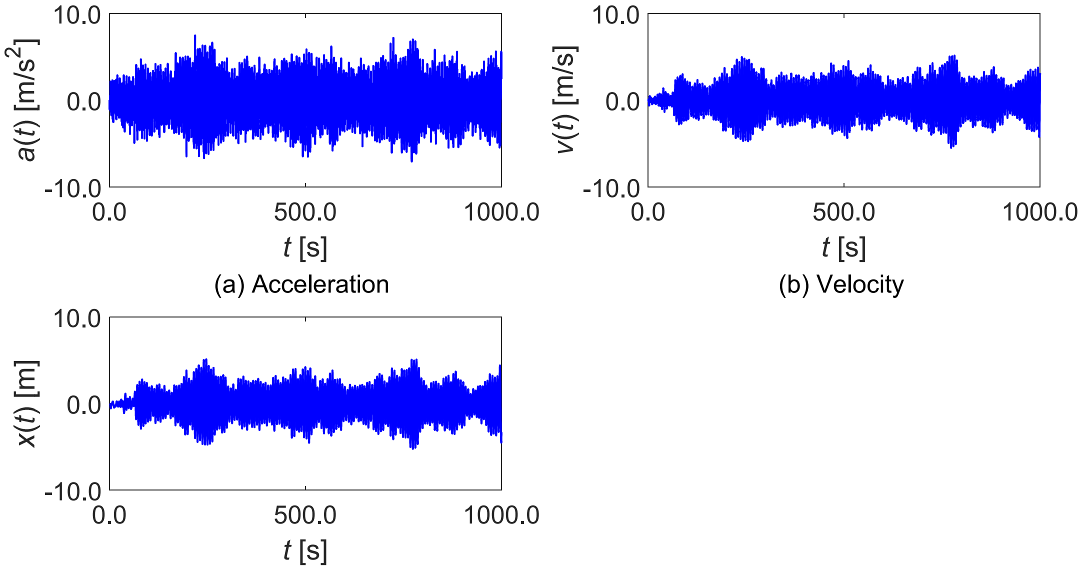
Using the above formulation of the state space representation, estimation of parameters using the proposed KF-SDA was conducted based on the algorithmic procedure shown in Fig. 1.

3.2 Results of the parameter estimation

An experiment was conducted for estimating the spring constant and the damping constant. The displacement, velocity and acceleration values were obtained by solving the equation of motion (Eq. (7)) using the Runge-Kutta method. The calculation conditions were as follows: the spring constant was $k = 1$; the damping constant was $c = 0.01$; the variance of the white noise was $\sigma_v^2 = 1$; and time step for the Runge-Kutta method was $\Delta t = 0.1$. In addition, the initial conditions were $x(0) = 0$ and $v(0) = 0$. Figs. 2(a)-(c) show the calculation results for the random oscillation response, where (a) shows the acceleration, (b) shows the velocity, and (c) shows the displacement.

The state estimation conditions were $R = 10^3$ and $D = 0.05$. Furthermore, the initial conditions for each moment were as follows: $k(0)$ and $c(0)$ were random numbers drawn from a Gaussian distribution, and $P(0) = I$. Fig. 3 shows the estimation results for the unknown parameters, where (a) shows the results for the spring constant, and (b) shows the result for the damping constant. In Fig. 3, the blue solid lines denote the estimation results obtained using the proposed KF-SDA, while the black dashed lines denote the actual (true) values. The estimated values obtained using the proposed KF-

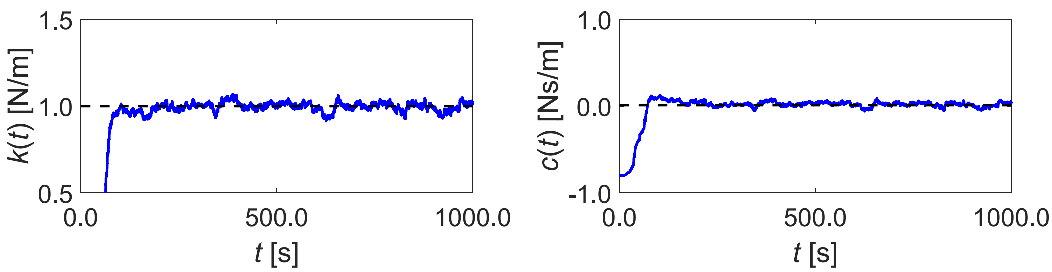
SDA converge to the actual (true) values. The estimated values for both the spring constant and the damping constant agreed well with the actual (true) values.



(a) Acceleration (b) Velocity

(c) Displacement

Fig. 2. Random vibration responses of a single-DOF system subjected to white noise excitation.



(a) Spring constant (b) Damping constant

Fig. 3. Parameter estimation results for the spring constant and the damping coefficient, obtained using the proposed KF-SDA, for a single-DOF system subjected to white noise excitation.

4. Applications to weakly nonlinear systems

4.1 Third-order nonlinear system subjected to white noise excitation

4.1.1 Mathematical formulation

The governing equation of a third-order nonlinear system for a unit mass, subjected to white noise excitation is as follows:

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx + k_3x^3 = w, \tag{10}$$

where x denotes the displacement, c is the damping coefficient, k is the spring constant, k_3 is the third-order nonlinear constant, and w denotes the white noise excitation force. Similar to the single-DOF system, we assumed displacement, velocity, and acceleration as signals. The state-space representation of this parameter estimation problem is as follows:

State equation

$$\frac{d\boldsymbol{\theta}(t)}{dt} = \boldsymbol{\gamma} v(t), \boldsymbol{\theta}(t) = \begin{bmatrix} k(t) \\ c(t) \\ k_3(t) \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{11}$$

Observation equation

$$\ddot{x}(t) = \boldsymbol{c}(t)\boldsymbol{\theta}(t) + w(t), \boldsymbol{c}(t) = [-x(t) \quad -\dot{x}(t) \quad -x(t)^3] \tag{12}$$

Using the above formulation of the state space representation, estimation of parameters using the proposed KF-SDA was conducted based on the algorithmic procedure shown in Fig. 1.

4.1.2 Results of the parameter estimation

An experiment aiming to estimate the spring constant, the third-order nonlinear spring constant, and the damping constant was conducted. The displacement, velocity and acceleration values were obtained by solving the equation of motion (Eq. (10)) using the Runge-Kutta method. The calculation conditions were as follows: the spring constant was $k = 1$; the damping constant was $c = 0.01$; the third-order nonlinear coefficient was $k_3 = 0.06$; the variance of the white noise was $\sigma_v^2 = 1$; the time step for the Runge-Kutta method was $\Delta t = 0.1$. In addition, the initial conditions were $x(0) = 0$ and $v(0) = 0$.

Fig. 4(a)-(c) show the calculation results for random oscillation response, where (a) shows the acceleration, (b) shows the velocity, and (c) shows the displacement.

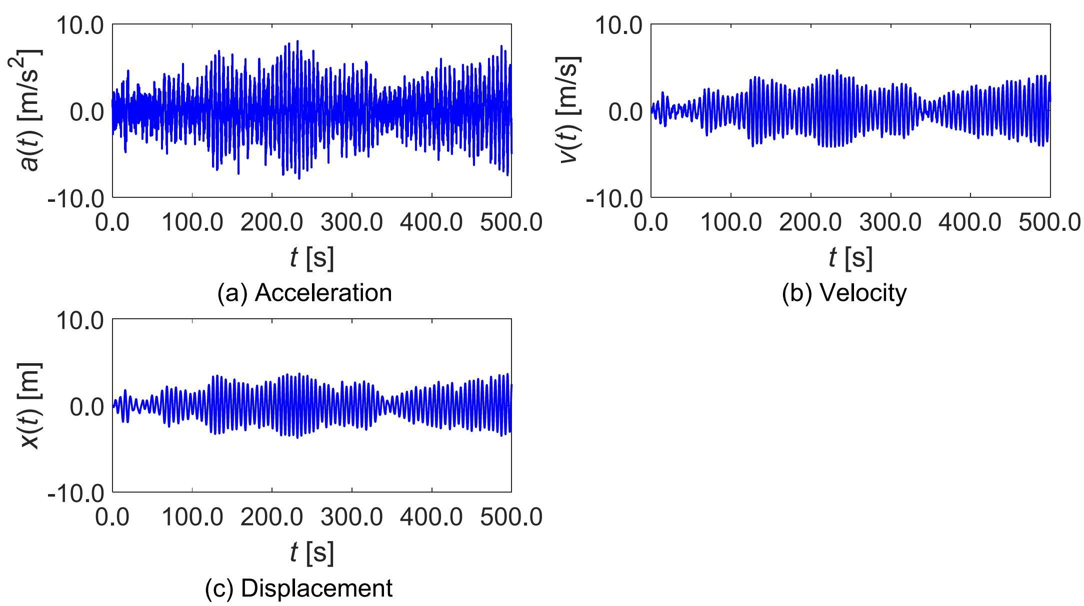
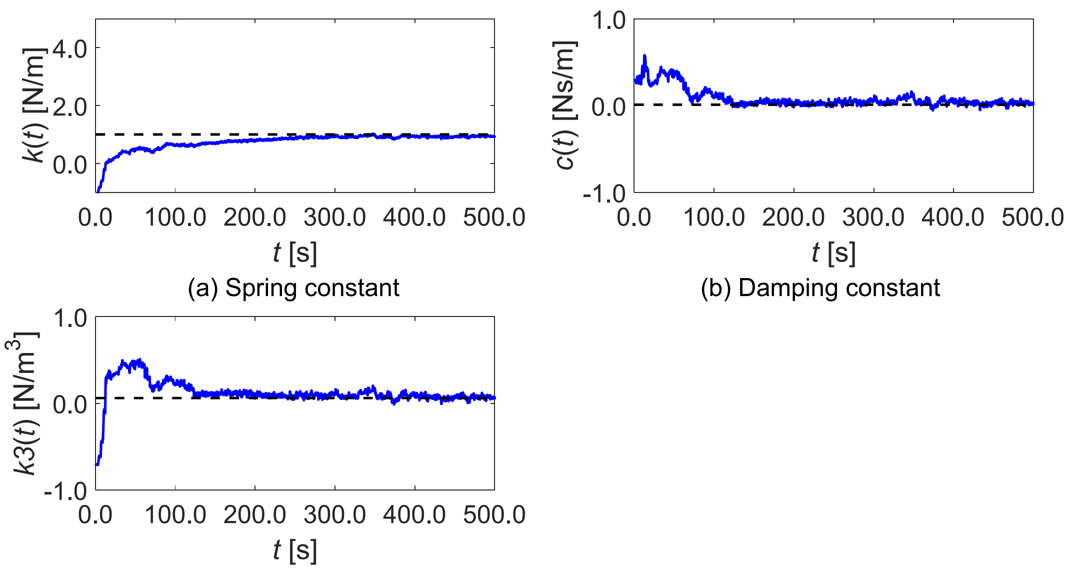


Fig. 4. Random vibration responses of a third-order nonlinear system subjected to white noise excitation.

The state estimation conditions were $R = 10^2$ and $D = 0.05$. Furthermore, the initial conditions for each moment were as follows: $k(0)$, $c(0)$, and $k_3(0)$ were random numbers drawn from a Gaussian distribution, and $P(0) = I$. Fig. 3 shows the estimation results for the unknown parameters, where (a) shows the results for the spring constant, (b) shows the results for the damping constant, and (c) shows the results for the third-order nonlinear spring coefficient. Here, the blue solid lines denote the estimation results obtained using the proposed KF-SDA, and the black dashed lines denote the actual (true) values. The estimated values obtained using the proposed KF-SDA converge to the actual (true) values. The estimated values of both the spring constant and damping constant agreed well with the true values.



(a) Spring constant (b) Damping constant

(c) Third-order nonlinear constant

Fig. 5. Parameter estimation results for the spring constant, the damping coefficient and the third-order nonlinear spring constant, obtained using the proposed KF-SDA.

4.2 Second-order nonlinear system subjected to white noise excitation

4.2.1 Mathematical formulation

The governing equation of a second-order nonlinear system for a unit mass subjected to white noise excitation is as follows:

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx + k_2x^2 = w, \tag{13}$$

where x denotes the displacement, c is the damping coefficient, k is the spring constant, k_2 is the second-order nonlinear constant, and w denotes the white noise excitation force. Here, similar to the single-DOF system, we assumed displacement, velocity and acceleration as signals. The state-space representation of this parameter estimation problem is as follows:

<u>State equation</u>

$$\frac{d\boldsymbol{\theta}(t)}{dt} = \boldsymbol{\gamma}v(t), \boldsymbol{\theta}(t) = \begin{bmatrix} k(t) \\ c(t) \\ k_2(t) \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{14}$$

<u>Observation equation</u>

$$\ddot{x}(t) = \boldsymbol{c}(t)\boldsymbol{\theta}(t) + w(t), \boldsymbol{c}(t) = [-x(t) \quad -\dot{x}(t) \quad -x(t)^2] \tag{15}$$

Using the above formulation of the state space representation, estimation of parameters was accomplished using the KF-SDA, based on the algorithmic procedure in Fig. 1.

4.2.2 Results of the parameter estimation

An experiment was conducted for estimating the spring constant, the second-order nonlinear spring constant, and the damping constant. The displacement, velocity and acceleration values were obtained by solving the equation of motion (Eq. (13)) using the Runge-Kutta method. The calculation conditions were as follows: the spring constant was $k = 1$; the damping constant was $c = 0.01$; the third-order nonlinear coefficient was $k_2 = 0.05;$ the variance of the white noise was $\sigma_v^2 = 1$; and the time step of the Runge-Kutta method was $\Delta t = 0.1$. In addition, the initial conditions were $x(0) = 0$ and $v(0) = 0$. Figs. 6(a)-(c) show the calculation results for the random oscillation response, where (a) shows the acceleration, (b) shows the velocity, and (c) shows the displacement.

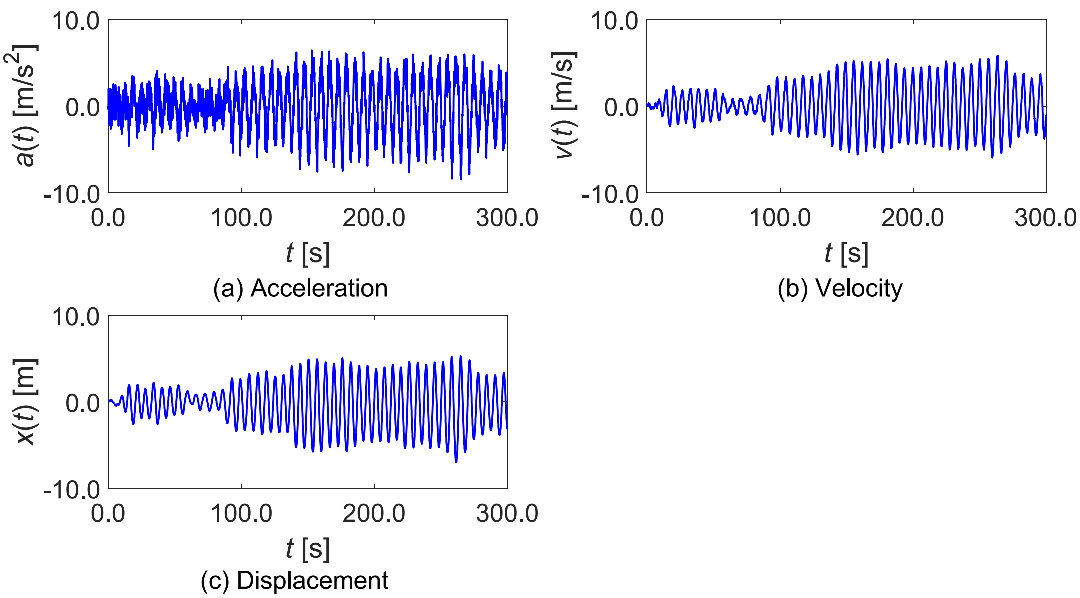
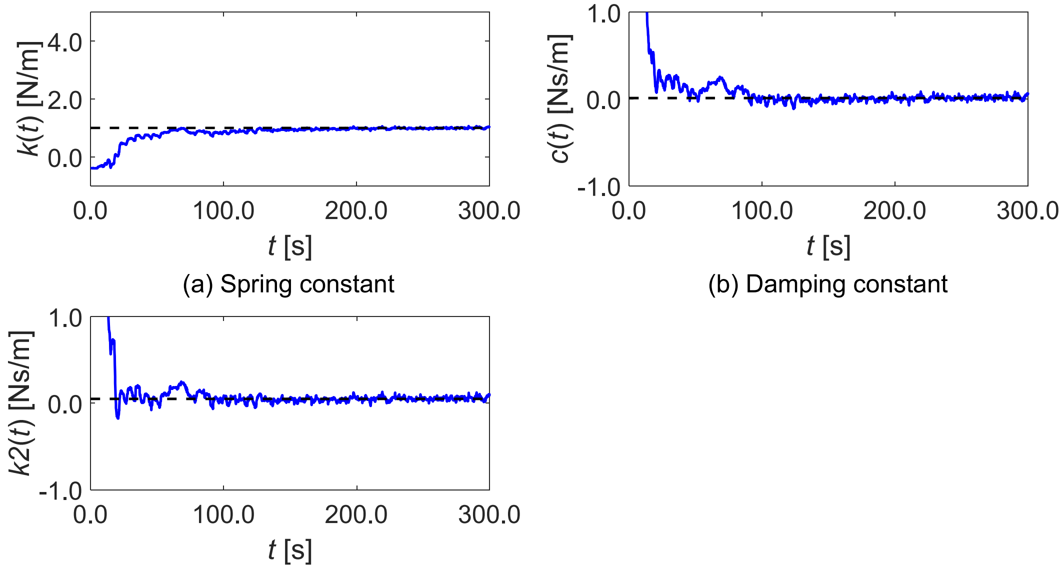


Fig. 6. Random vibration responses for a second-order nonlinear system subjected to white noise excitation.

The state estimation conditions were $R = 10^2$ and $D = 0.05$. Furthermore, the initial conditions for each moment were as follows: $k(0)$, $c(0)$ and $k_2(0)$ were random numbers drawn from a Gaussian distribution, and $P(0) = I$. Fig. 3 shows the estimation results for the unknown parameters, where (a) shows the results for the spring constant, (b) shows the results for the damping constant, and (c) shows the results for the second-order nonlinear spring coefficient. Here, the blue solid lines denote the estimation results obtained using the proposed KF-SDA, and the black dashed lines denote the actual (true) values. The estimated values using the proposed KF-SDA converged to the actual (true) values. The estimated values of both the spring constant and damping constant agreed well with the actual values.



(a) Spring constant (b) Damping constant

(c) Second-order nonlinear constant

Fig. 7. Parameter estimation results for the spring constant damping coefficient and the second-order nonlinear spring constant using the proposed KF-SDA.

5. Application to a single-DOF system subjected simultaneously to white noise and random parametric excitations

5.1 Mathematical formulation

Here, we consider the application of the proposed method to a single-DOF system subjected simultaneously to white noise and random parametric excitations. The governing equation of a single-DOF system for a unit mass subjected simultaneously to random parametric and white noise excitations is

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + (k + w_1)x = w_2, \tag{16}$$

where x represents the displacement, c is the damping coefficient, k is the spring constant, w_1 is the white random parametric excitation force, and w_2 represents the white noise excitation force. The state-space representation of this parameter estimation problem is as follows:

<u>State equation</u>

$$\frac{d\boldsymbol{\theta}(t)}{dt} = \boldsymbol{\gamma} v(t), \boldsymbol{\theta}(t) = \begin{bmatrix} X(t) \\ c(t) \end{bmatrix}, X(t) = k(t) + w_1(t), \boldsymbol{\gamma} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{17}$$

<u>Observation equation</u>

$$\ddot{x}(t) = \boldsymbol{c}(t)\boldsymbol{\theta}(t) + w(t), \boldsymbol{c}(t) = [-x(t) \quad -\dot{x}(t)] \tag{18}$$

Here, a new stochastic variable $X(t)$ is introduced into the state equation. In the prediction step, the moment equation for first-order is as follows:

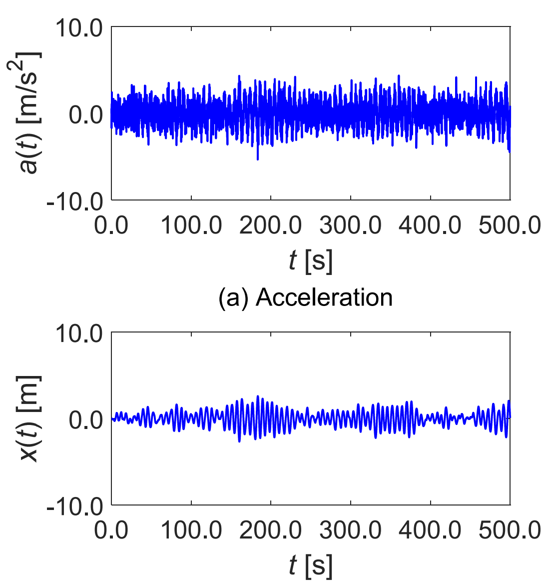
$$\frac{d\mathrm{E}[X(t)]}{dt} = 0, \qquad \frac{d\mathrm{E}[c(t)]}{dt} = 0. \tag{19}$$

In Eq. (19), the right-hand-term of the first equation is rewritten using the calculation rule of the expected value, as $\mathrm{E}[X(t)] = \mathrm{E}[k(t)] + \mathrm{E}[w_1(t)] = \mathrm{E}[k(t)]$. Here, the mean value of $w_1(t)$ was assumed to be zero. As a result, the moment equation (Eq. (19)) describes a linear single-DOF system subjected to white noise. Therefore, the estimation of unknown parameters using the proposed KF-SDA is the same as for a single-DOF system subjected simultaneously to white noise and random parametric excitations.

Using the above formulation of the state space representation, estimation of parameters using the proposed KF-SDA was conducted based on the algorithmic procedure shown in Fig. 1.

5.2 Results of the parameter estimation

An experiment was conducted for estimating the spring constant and damping constant. The displacement, velocity, and acceleration values were obtained by solving the equation of motion (Eq. (16)), using the Runge-Kutta method. The calculation conditions were as follows: the spring constant was $k = 1$; the damping constant was $c = 0.05$; the variance of the parametric excitation noise was $\sigma_1^2 = 0.1$; the variance of the observation white noise was $\sigma_2^2 = 1$; and the time step of the Runge-Kutta method was $\Delta t = 0.1$. In addition, the initial conditions were $x(0) = 0$ and $v(0) = 0$. Figs. 8(a)-(c) show the calculation results for the random oscillation response, where (a) shows the acceleration, (b) shows the velocity, and (c) shows the displacement.



(a) Acceleration (b) Velocity (c) Displacement

Fig. 8. Random vibration responses for a single-DOF system subjected simultaneously to white noise and random parametric excitations.

The state estimation conditions were $R = 10^2$ and $D = 0.05$. Furthermore, the initial conditions for each moment were as follows: $k(0)$ and $c(0)$ were random numbers drawn from a Gaussian distribution, and $P(0) = I$. Fig. 9 shows the estimation results of the unknown parameters, where (a) shows the results for the spring constant, and (b) shows the results for the damping constant. Moreover, the blue solid lines denote the estimation results obtained using the proposed KF-SDA, while the black dashed lines show the actual (true) values. The estimated values obtained using the proposed KF-SDA converge to the actual (true) values. The estimated values of both the spring constant and damping constant agreed well with the true values.

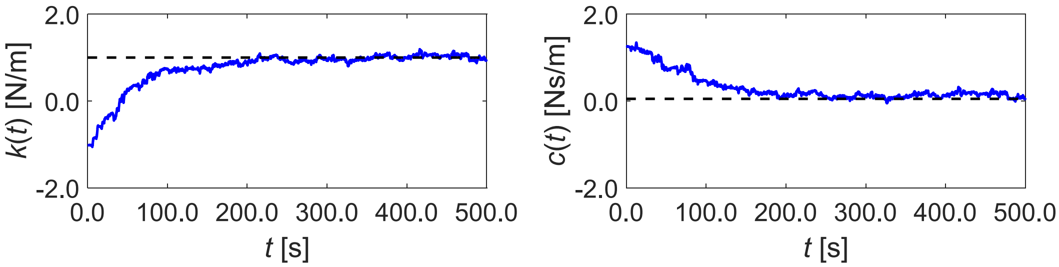


Fig. 9. Parameter estimation results for the spring constant and damping coefficient, obtained using the proposed KF-SDA.

6. Future work

The method for estimation of unknown parameters that we proposed in the present study assumes a large number of signals. In a typical application, however, only a limited amount of data (from a small number of sensors) is available for analysis. Thus, future studies will seek to generalize the currently proposed method to cases with a small number of sensors.

7. Conclusions

In this study, the modified Kalman filter analysis approach (KF-SDA) that uses the solution of the moment equation in the prediction step was expanded to address the parameter estimation problem. Consequently, the following results were obtained.

(1) The modified Kalman filter with stochastic dynamic analysis in the prediction step was expanded to address the parameter estimation problem. The parameter estimation problem was formulated by assuming that the dynamics of the unknown parameters in the system can be described by the Brownian motion.

(2) The fundamental operation was validated using the stochastic responses of a single-DOF vibratory system subjected to white noise. The resultant estimated parameters (i.e., the spring constant and the damping constant) agreed well with the actual (true) values.

(3) Application of the proposed method to a weakly nonlinear KF-SDA system was demonstrated. We considered the cases of third-order and second-order nonlinearities on behalf of symmetrical and asymmetrical nonlinearities, respectively. The resultant estimated parameters (i.e., the spring constant, the nonlinear spring constant, and the damping constant) agreed well with the actual (true) values.

(4) Application of the proposed method to a single-DOF system described by a unit mass subjected simultaneously to random parametric and white noise excitations was considered. The parameter estimation problem was formulated with an additional stochastic variable, which was the sum of the spring constant and random parametric excitation. The resultant estimated parameters (i.e., the spring constant and the damping constant) agreed well with the actual (true) values.

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