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Frequency Estimation Sampling Circuit with Hilbert Filter and Proactive Usage of Aliasing Phenomenon

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Abstract. This paper describes a high-frequency signal estimation circuit using an analog Hilbert filter and multiple low-frequency sampling circuits followed by analog-to-digital converters (ADCs); here the sampling frequencies are relatively prime. Our proposed waveform sampling system uses aliasing phenomena of sampling in frequency domain proactively, and the input signal frequency can be estimated based on the residue number theory. A conventional high frequency sampling circuit can directly estimate a high frequency input signal, but handling of high frequency signals in electronic circuits is difficult; on the other hand, the proposed circuit is relatively easy to implement. Cosine wave with high frequency is provided as an input signal, and then cosine and sine signals with the same frequency are generated with an analog Hilbert filter (such as an RC polyphase filter); they are fed into sampling circuits with different (relatively prime) and low sampling frequencies. Their analog outputs are analog-to-digital converted and for their digital outputs, complex FFTs are performed. Since the high frequency signal is sampled with low frequency clocks, the aliasing (spectrum folding) occurs. However, each aliased frequency is different from each other because each sampling clock frequency is different in sampling circuits. Then based on the Chinese remainder theorem, the input frequency can be estimated. Notice that usage of the analog Hilbert filter is new in this paper; if the analog Hilbert filter is not used and the cosine input is directly sampled, the residue frequency cannot be obtained.

1. Introduction

The authors have been conducting research on applying classical mathematics to electronic circuit designs [1]. For example, a magic square algorithm is used to select the order of the unit circuits in a 2-dimensional array for the unary-type DAC linearity improvement [2, 3], and a Fibonacci sequence is used for the redundant design to improve the reliability of successive approximation analog-to-digital converter [4]. In this paper, we examine the method of the Chinese remainder theorem and the residue system algorithm written in the Chinese mathematical classic text book by the Chinese scholar of Sun Zu, for the frequency estimation of the sinusoidal signal waveform. The proposed circuit can estimate the high frequency signal, from multiple low frequency sampling circuits, combined with an analog Hilbert filter [5] (which is new), but without a very high frequency sampling circuit which is difficult to realize. Its principle and circuit configuration as well as simulation verification are shown.

2. Chinese Remainder Theorem

The following problems have been written in the ancient Chinese mathematics book 'Sun Tzu calculation':
"When an integer number is divided by 3, its residue is 2, divided by 5, its residue is 3, and divided by 7, its residue is 2. What is the original number?" The answer is 23.

Generalization of this problem is the "Chinese Remainder Theorem." This Sun Tzu calculation was transferred as "one hundred and five subtraction problem" to Japan, and now the Chinese remainder theorem has been proved by Euclidean algorithm in modern mathematics.

An example of the residue number system is given as follows:
Consider the natural numbers 2, 3, 5 of relatively prime. Notice $N = 2 \times 3 \times 5 = 30$, and suppose that one of the integers from 0 to $N - 1$ (= 29) is k. Also the remainder of dividing k by 2 is m_1 and the remainder of dividing k by 3 is m_2, while the remainder of dividing k by 5 is m_3. The set of k and (m_1, m_2, m_3) corresponds one to one (Table 1). The Chinese Remainder Theorem is an algorithm for finding k from (m_1, m_2, m_3). The details of the residue number system are described in [6].

This residue number system has been widely used in digital arithmetic circuit design, and we have used in mixed-signal circuit (time-to-digital converter) design [7]. In this paper, we use it for the waveform sampling system.

Table 1. Correspondence between an integer k and its residue representation (m_1, m_2, m_3)

m_1	m_2	m_3	k
0	0	0	0
1	1	1	1
2	2	2	2
0	3	3	3
1	4	4	4
2	0	5	5
0	1	6	6
1	2	0	7
2	3	1	8
0	4	2	9
1	0	3	10
2	1	4	11
0	2	5	112
1	3	6	13
2	4	0	14

m_1	m_2	m_3	k
0	0	1	15
1	1	2	16
2	2	3	17
0	3	4	18
1	4	5	19
2	0	6	20
0	1	0	21
1	2	1	22
2	3	2	23
0	4	3	24
1	0	4	25
2	1	5	26
0	2	6	27
1	3	0	28
2	4	1	29

3. Residue number sampling system

3.1 Sampling theorem and aliasing phenomenon

Waveform sampling is a technique for converting a continuous analog signal into a discrete-time signal [8, 9]. When the analog signal $f(t)$ of the input frequency f_{in} is sampled at the time interval T, the sampling frequency f_s is represented by $1 / T$. Fig. 1 shows its example; Fig.1 (a) shows an analog signal while Fig. 1 (b) shows its sampled signal. The pulsed discrete signal after sampling is called as a sampled function $g_s(t)$. If f_s is higher than twice the maximum frequency of $f(t)$, the original analog signal $f(t)$ can be completely reconstructed from $g_s(t)$. This is known as the sampling theorem.

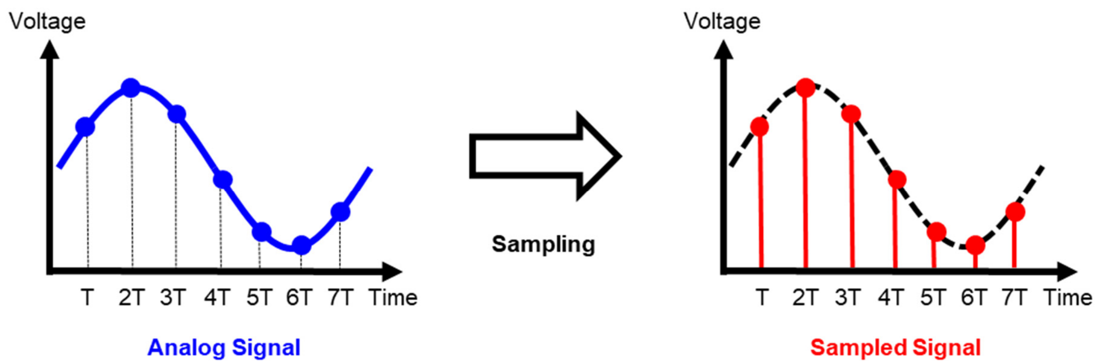


Fig. 1. Waveform sampling

If f_s is lower than or equal to twice the maximum frequency of $f(t)$, f_{in} cannot be distinguished from different continuous signals. For example, when sine waveforms with frequencies of 1 kHz and 7 kHz are sampled at 8 kHz, both sampling points are completely identical and indistinguishable (Fig. 2). This phenomenon occurs when $f_1 + f_2 = m\, f_s$ or $|\, f_1 - f_2\,| = n\, f_s$ (m and n are integers); in Fig, 2, $f_1 = 1$ kHz, $f_2 = 7$ kHz, and $f_s = 8$ kHz with $m = n = 1$. This is called as the aliasing phenomenon, and here we use it proactively in our proposed sampling system.

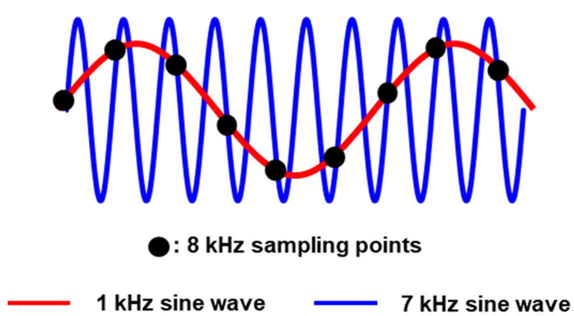


Fig. 2. Example of aliasing phenomenon

3.2 Waveform sampling and residue frequency

Figure 3 shows a waveform obtained by performing FFT at a sampling frequency of 8 kHz with a sine wave having a frequency of 30 kHz. Since this does not satisfy the sampling theorem condition, within the sampling frequency band, the spectrum appears to be folded at 2 kHz and 6 kHz symmetrically at the Nyquist frequency (8 kHz / 2 = 4 kHz). The frequency 6 kHz is the residue when the input frequency 30 kHz is divided by the sampling frequency of 8 kHz, and hence 6 kHz is interpreted as the residue frequency. Thus we remove the 2 kHz spectrum component so that we extract and use only the 6 kHz spectrum component here.

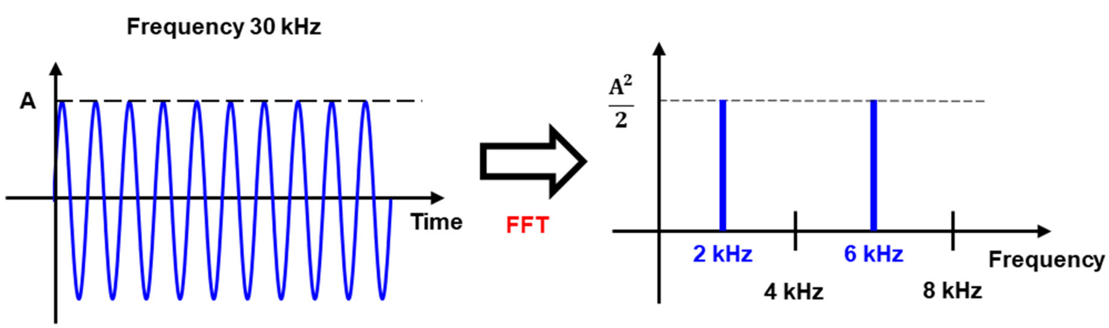


Fig. 3. 30 kHz sine wave sampled at 8 kHz for FFT

The same spectrum as in Fig. 3 can be obtained by sampling a cosine wave with a frequency of 30 kHz at 8 kHz and performing an FFT.

A waveform to which a complex FFT is performed by multiplying the sine wave with a frequency of 30 kHz by 'j' which is an imaginary unit is shown in Fig. 3. In the waveform of Fig. 4, the positive and negative of the 2 kHz spectrum appear to be inverted (with 180° phase shift). The inverted spectrum is not a residue frequency. Therefore, by adding the spectrum of the cosine wave in Fig. 3 and the spectrum of the sine wave multiplied by the imaginary unit 'j' in Fig. 4, only the spectrum of 6 kHz (which is the residue frequency) can be taken out as shown in Fig. 5.

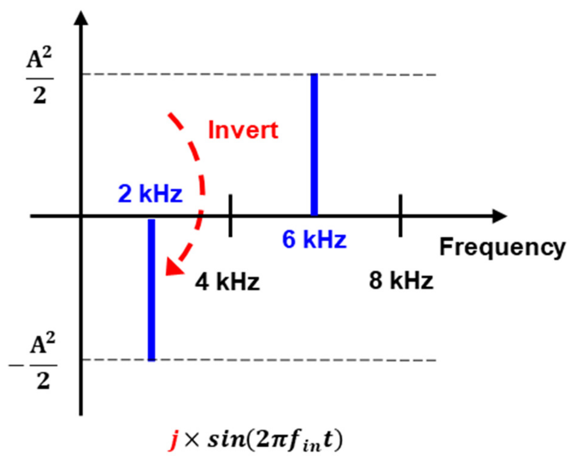


Fig. 4. FFT of j × sin(2π$f_{in}t$) (f_{in}: 30 kHz, sampling frequency: 8 kHz)

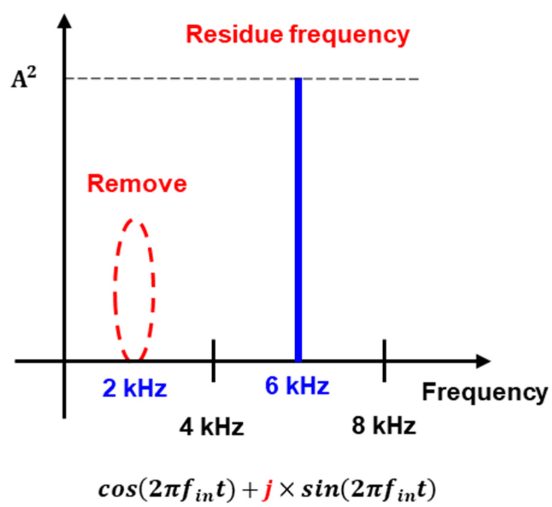


Fig. 5. FFT of cos(2π$f_{in}t$) + j × sin(2π$f_{in}t$) (f_{in}: 30 kHz, sampling frequency: 8 kHz)

4. Proposed waveform sampling system for frequency estimation

4.1 RC polyphase filter

Here we consider an RC polyphase filter as an analog Hilbert filter, which can generate cosine and sine waves from a cosine wave (Fig. 6) [2]. Usage of the analog Hilbert filter in front of sampling circuits are new, compared to the circuit [1].

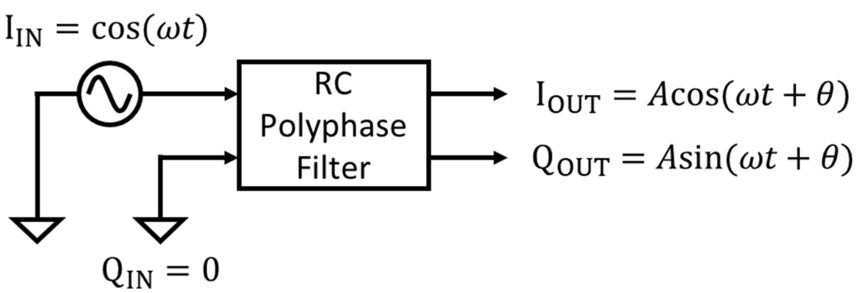


Fig. 6. RC polyphase filter

4.2 Proposed frequency estimation circuit

The proposed frequency estimation circuit is shown in Fig. 7. First, a sinusoidal signal whose frequency is unknown is input to the RC polyphase filter, which generates in-phase and quadrature signals of the same frequency. Next, the generated signals are provided to three pairs of sampling circuits. Sampling frequencies for each pair need to be relatively prime. Then the residue frequency is determined by performing complex FFT on the output of the sampling circuit pair. Finally, the input frequency is estimated by the obtained residue frequency and the residue theorem [10].

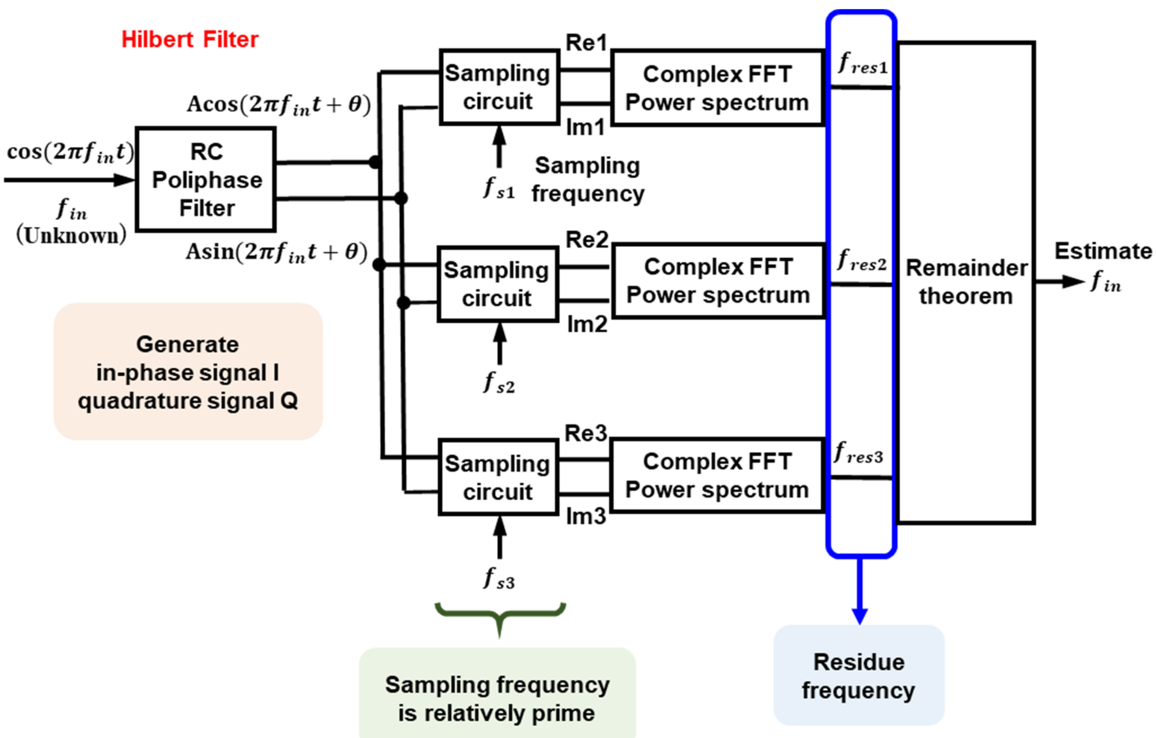


Fig. 7. Proposed frequency estimation system

4.3 Simulation verification of the proposed circuit

Simulation settings:

- ·Input frequency: 12 GHz
- ·Sampling frequency: 229 kHz, 233 kHz, 239 kHz
- ·Frequency resolution: 1 kHz

The simulation was performed with the above settings. The frequency measurement range is determined by the product of sampling frequencies, which is about 20 GHz obtained from the product of 229 kHz, 233 kHz, and 239 kHz in this case. The frequency measurement up-to about 20 GHz is possible using sampling frequencies of about 200 kHz. Further, a wider frequency measurement range can be obtained with the increase of the number of sampling circuits. Fig. 8 shows the simulation results where an IQ signals are generated by inputting a cosine wave to the RC polyphase filter, where the top graph in blue shows the I-channel input signal, the middle graph in blue shows the I-channel output signal and the bottom graph shows the Q-channel output signal; the IQ output signals have a 90-degree phase difference. Notice that the Q-channel input signal is zero, i.e., its terminal is connected to the ground.

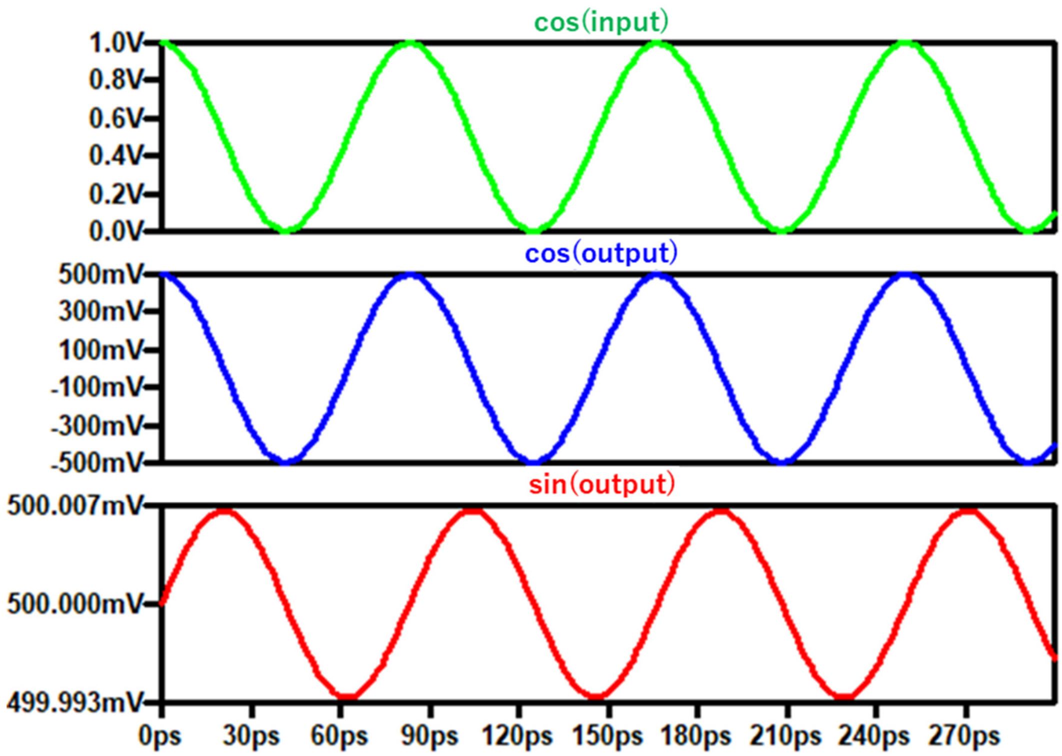


Fig. 8. Waveform generation by RC polyphaser filter. The top waveform in blue shows the I-channel signal, and the middle in blue shows the I-channel output signal, while the bottom shows the Q-channel output signal

Fig. 9 shows the simulation results of sampling the obtained IQ signals at each sampling frequency. The results obtained by the complex FFT are shown in Fig. 10. The frequencies at the peak power in three spectrums are 171 kHz for 229 kHz sampling, 34 kHz for 233 kHz sampling, and 49 kHz for 239 kHz sampling, respectively; thus the residue frequencies are 171 kHz, 34 kHz, and 49 kHz, respectively. Table 2 shows a part of the remainder theorem, and we see that when the residue frequencies are 171 kHz, 34 kHz and 49 kHz, the input frequency can be uniquely determined to be 12 GHz. Fig. 11 shows the result obtained by simulation.

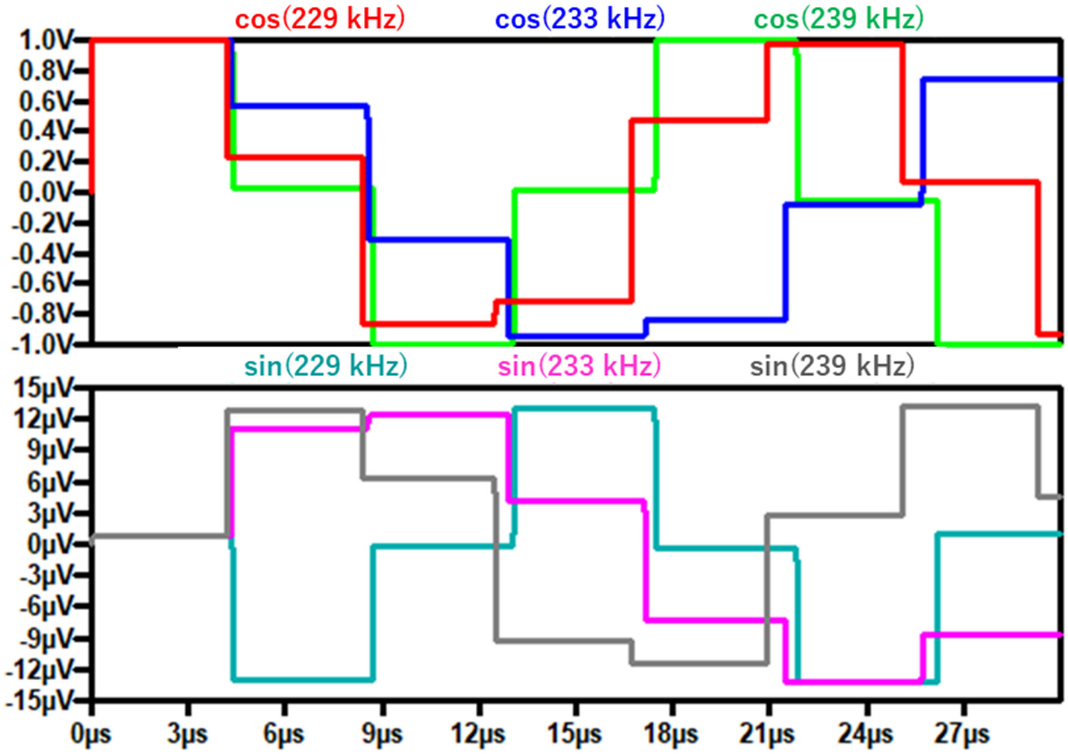
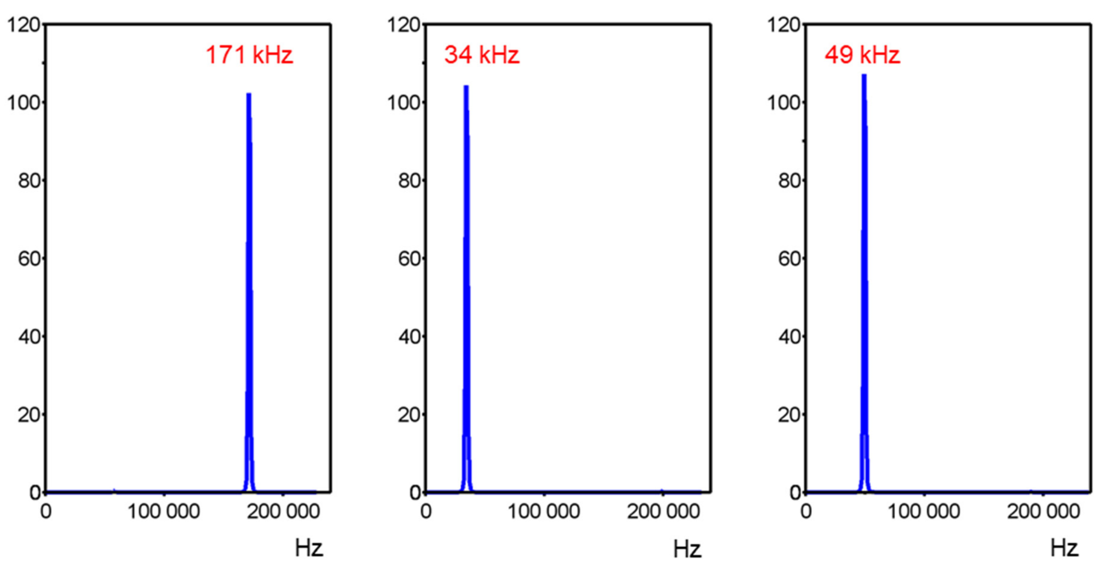


Fig. 9. Waveforms after sampling at each sampling frequency. In the top graph, the red shows the 171 kHz cosine wave with 229 kHz sampled, the blue is 34 kHz one with 233 kHz, and the green is 49 kHz one with 239 kHz, respectively. In the bottom graph, the dark green shows the 171 kHz sine wave with 229 kHz sampled, the purple is 34 kHz one with 233 kHz, and the brown is 49 kHz one with 239 kHz, respectively



(a) 229 kHz sampling (b) 233 kHz sampling (c) 239 kHz sampling

Fig. 10. Complex FFT result

Table 3. Residues for m_1, m_2, m_3 and original number k for error frequency verification

m_1	m_2	m_3	k
0	0	0	0
1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
169	32	47	11999998
170	33	48	11999999
171	34	49	12000000
172	35	50	12000001
173	36	51	12000002
⋮	⋮	⋮	⋮
226	230	236	12752320
227	231	237	12752321
228	232	238	12752322

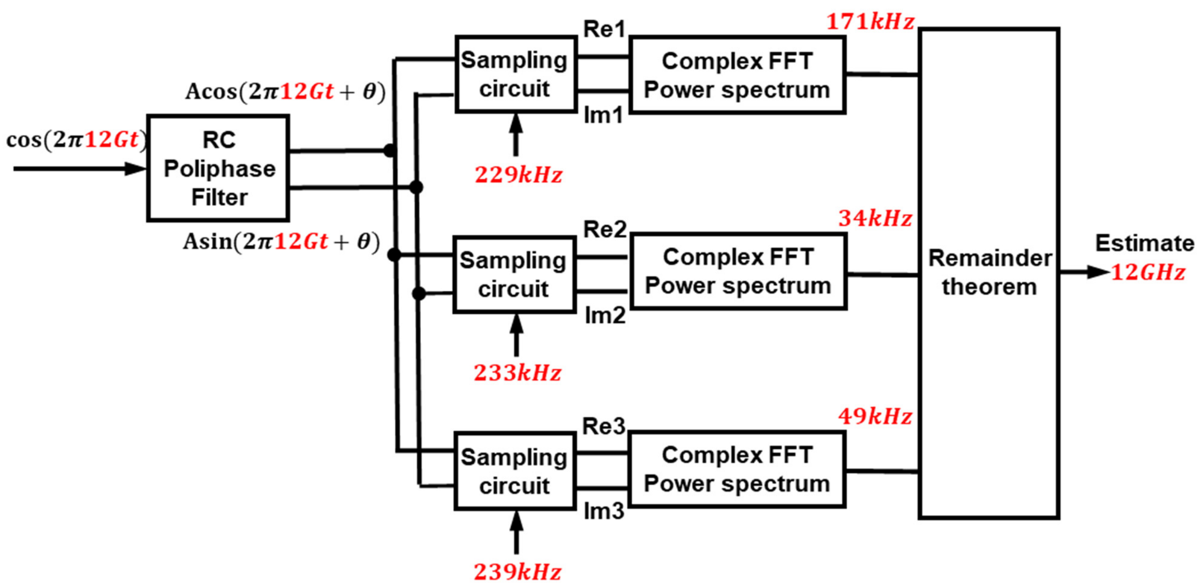
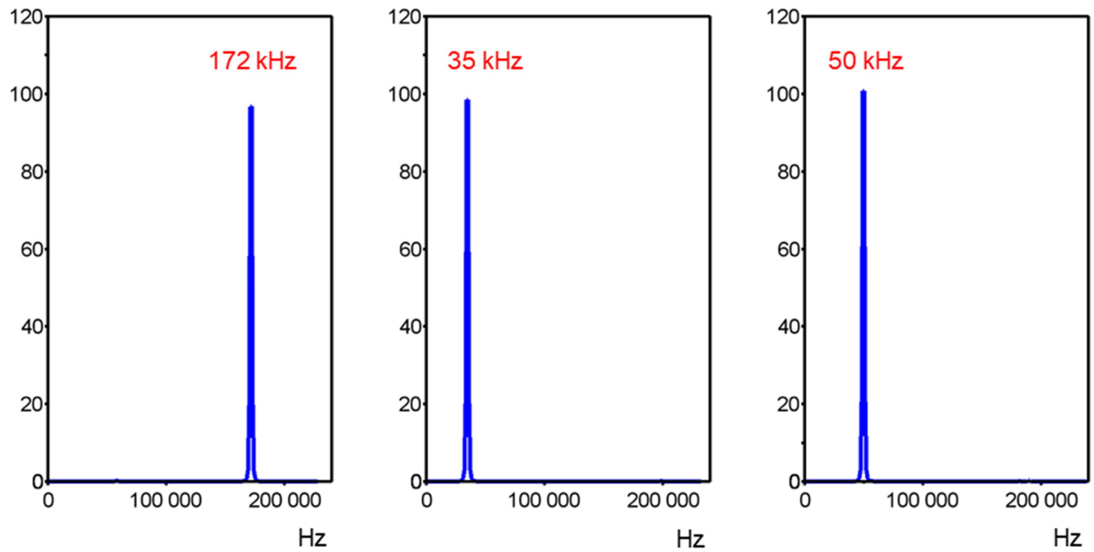


Fig. 11. Operation of the proposed sampling system

4.4 Simulation when the input signal has an error frequency finer than the frequency resolution

We have examined the case where the input signal has an error frequency finer than frequency resolution of 1 kHz. When the 12 GHz input signal is sampled at 229 kHz and the complex FFT is performed, the remainder of 171 kHz when 12 GHz is divided by 229 kHz is obtained as the residue frequency. Next, the input signal is 12 GHz + 0.1 kHz. In this case, the residue frequency is 171.1 kHz. Since the frequency resolution is 1 kHz, the frequency axis spacing is 1 kHz. Therefore, the result is plotted closer to 171 kHz and 172 kHz. The result in this case is 171 kHz. We see from these facts that the result is the same when the error frequency (f_e) is $-$ 0.5 kHz $\leq f_e <$ 0.5 kHz. Also, when the input signal of 12 GHz + 0.5 kHz is sampled at 229 kHz and complex FFT is performed, the residue frequency is 172 kHz. When a similar input signal is sampled at 233 kHz and 239 kHz, and a complex FFT is performed, the residue frequencies are 35 kHz and 50 kHz. Fig. 12 shows the results of this simulation. Using these residue frequencies and the remainder theorem, the input signal can be estimated to be 12 GHz + 1 kHz from Table 3. As a result, the estimated frequency can obtain an accurate value even when the input signal has an error frequency finer than the frequency resolution; this means that the proposed sampling system can estimate the input signal frequency in a stable and robust way.



(a) 229 kHz sampling (b) 233 kHz sampling (c) 239 kHz sampling

Fig. 12. Complex FFT result (Error frequency verification)

Table 3. Residues for m_1, m_2, m_3 and original number k for error frequency verification

m_1	m_2	m_3	k
0	0	0	0
1	1	1	1
2	2	2	2
⋮	⋮	⋮	⋮
169	32	47	11999998
170	33	48	11999999
171	34	49	12000000
172	35	50	12000001
173	36	51	12000002
⋮	⋮	⋮	⋮
226	230	236	12752320
227	231	237	12752321
228	232	238	12752322

5. Conclusion

We have proposed a waveform sampling circuit to estimate the frequency of a high frequency signal from multiple low frequency sampling circuits with an analog Hilbert filter, and confirmed by theoretical analysis and simulation. Since the measurement range of the input frequency is determined by the product of multiple sampling frequencies, the measurable range can be wide. Therefore, by increasing the number of sampling circuits, the measurable frequency becomes very wide [11].

Notice that several sampling circuits using the residue number system have been investigated such as [9], and the originality of this paper is usage of an analog Hilbert filter (RC polyphaser filter), and this can be one of candidates for such sampling circuits and one of applications for the RC polyphase filter. We have also confirmed that it is possible to estimate close frequency value even if the input signal frequency has some deviation.

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